ETH zürich

Validation of a Rheological Model for Non-Newtonian Fluid Flow

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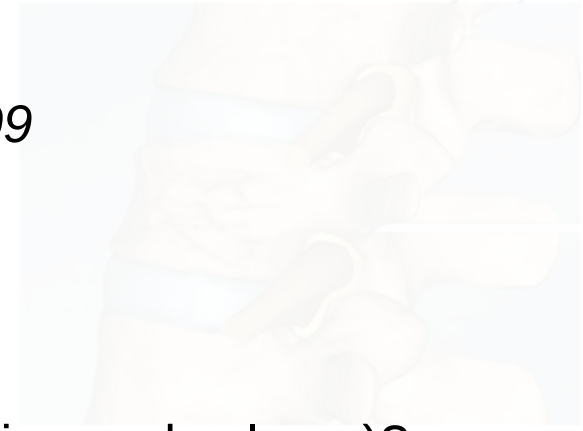
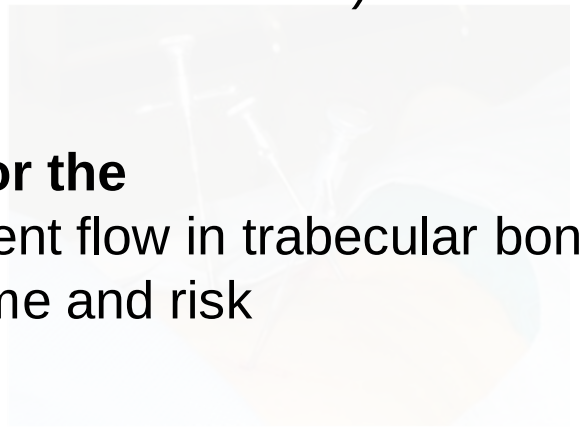
Percutaneous Vertebroplasty



- Procedure outcome is **controversial**:
  - *Buchbinder et al., N Engl J Med, 2009*
  - *Kallmes et al., N Engl J Med, 2009*
- Source of **uncertainty**:
  - Bone cement?
  - Treatment strategy (biomaterial location and volume)?
  - Experience of the clinician?

⇒ **Motivation: develop *in-silico* models for the**
  - Simulation and investigation of cement flow in trabecular bone
  - Optimization of the treatment outcome and risk



Vertebroplasty Effectiveness Assessment

Rheological Model for Non-Newtonian Fluid Flow in Vertebral Trabecular Bone

- Darcy flow:

$$\underline{q} = \frac{\underline{\underline{k}}^s}{\bar{\mu}} \nabla p$$

- Reynolds number $\overline{\mathrm{Re}}$:

$$\overline{\mathrm{Re}} = 4 \frac{\rho \left\| \underline{q} \right\|}{\bar{\mu} S_v}$$

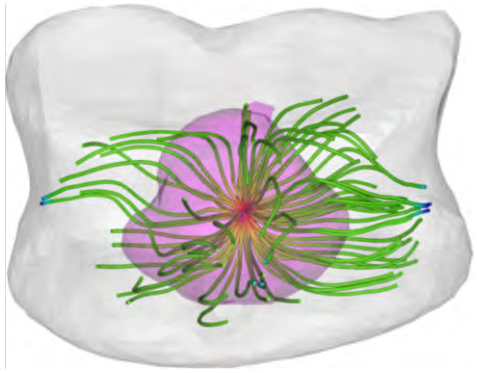
- Normalized Navier Stokes:

$$-\mathrm{Re} \nabla p' + \nabla^2 \underline{u}' = \mathrm{Re}\left(\underline{u}' \cdot \nabla\right) \underline{u}'$$
$$\nabla \cdot \underline{u}' = 0$$

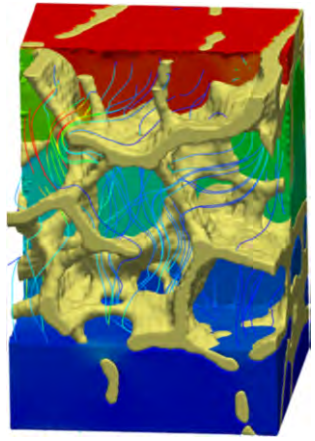
- Reynolds number Re:

$$\mathrm{Re} = \frac{\rho {u_0}^{2-n} D^n}{C}$$

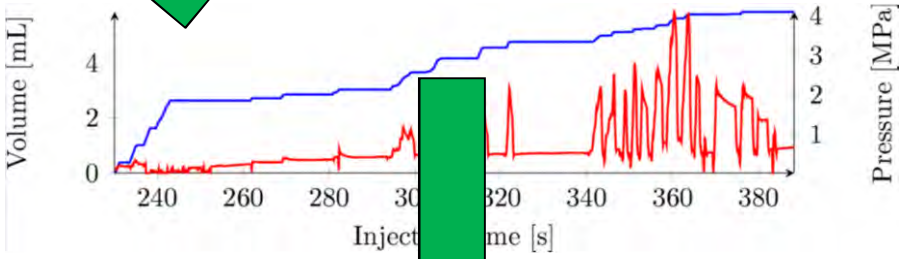
$$\overline{\mathrm{Re}} = \frac{\oiint \mathrm{Re}(x,y,z)\, dxdydz}{\oiint dxdydz}$$



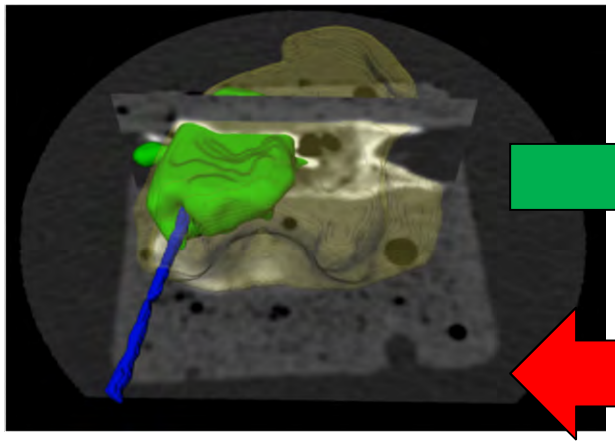
Slattery-Whitaker theorem
S. Whitaker et al., 1969

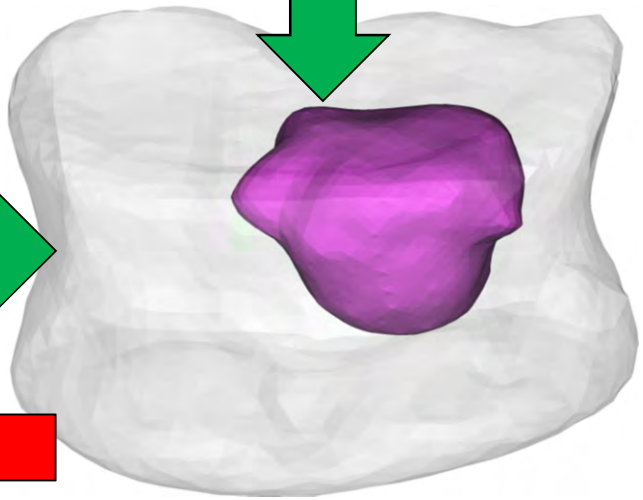


Experimental Validation

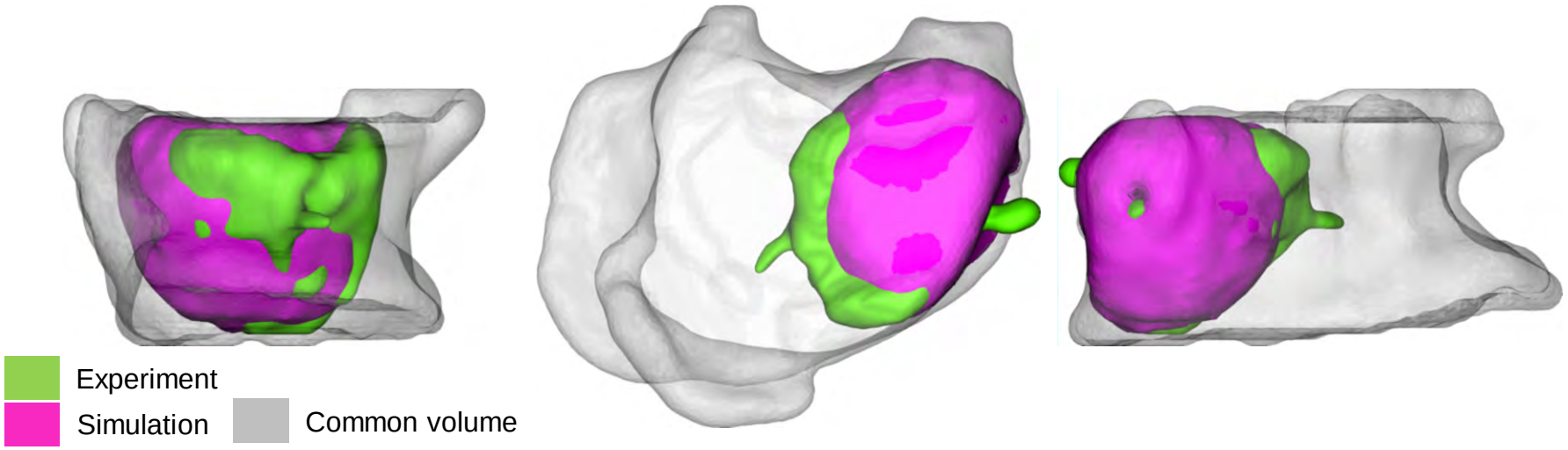


Experiments performed at the Istituto Ortopedico Rizzoli, Bologna, with the support of L. Cristofolini

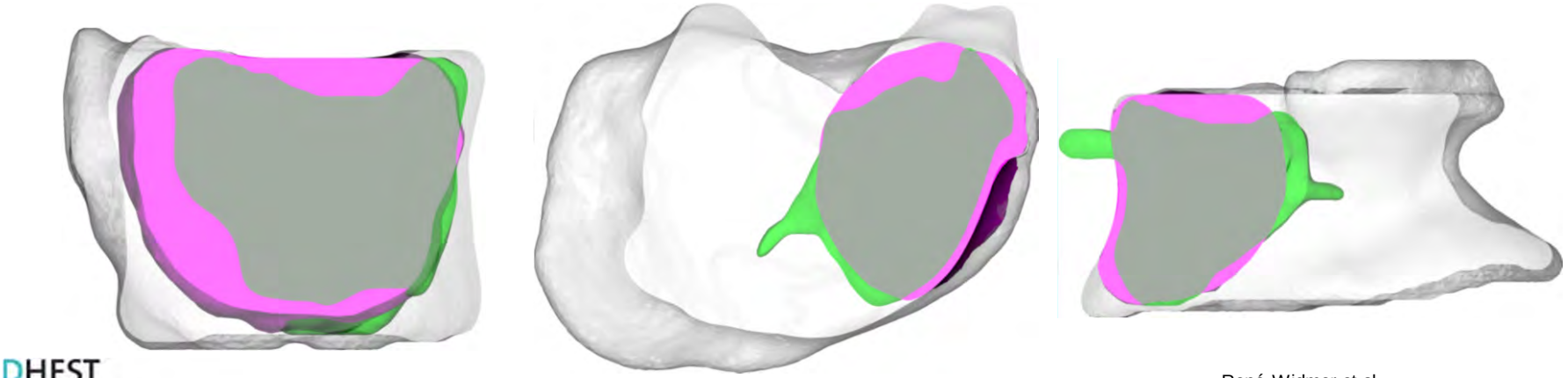




Experimental vs. Predicted Cement Pattern

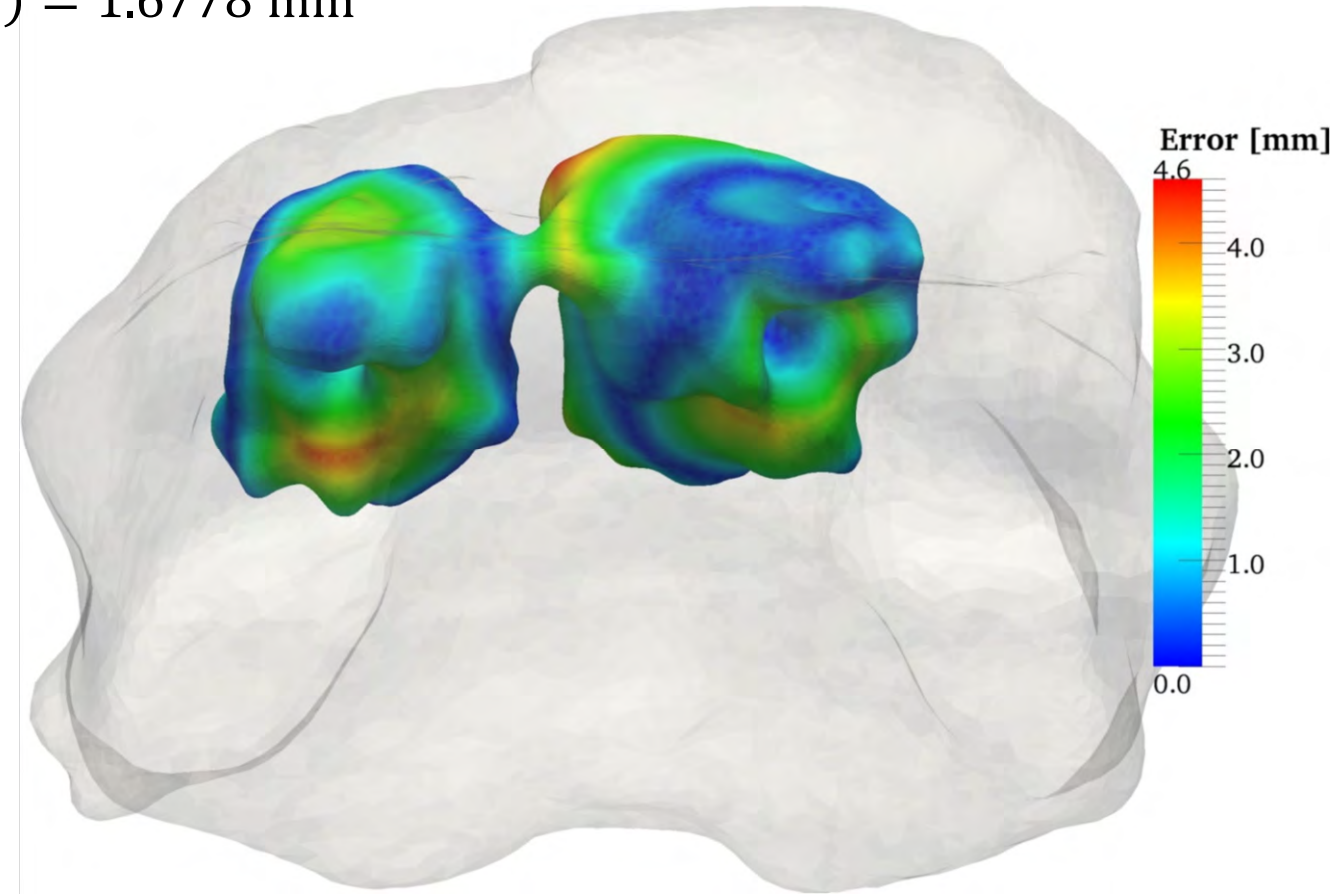


Saggital plane | Transerve plane | Coronal plane

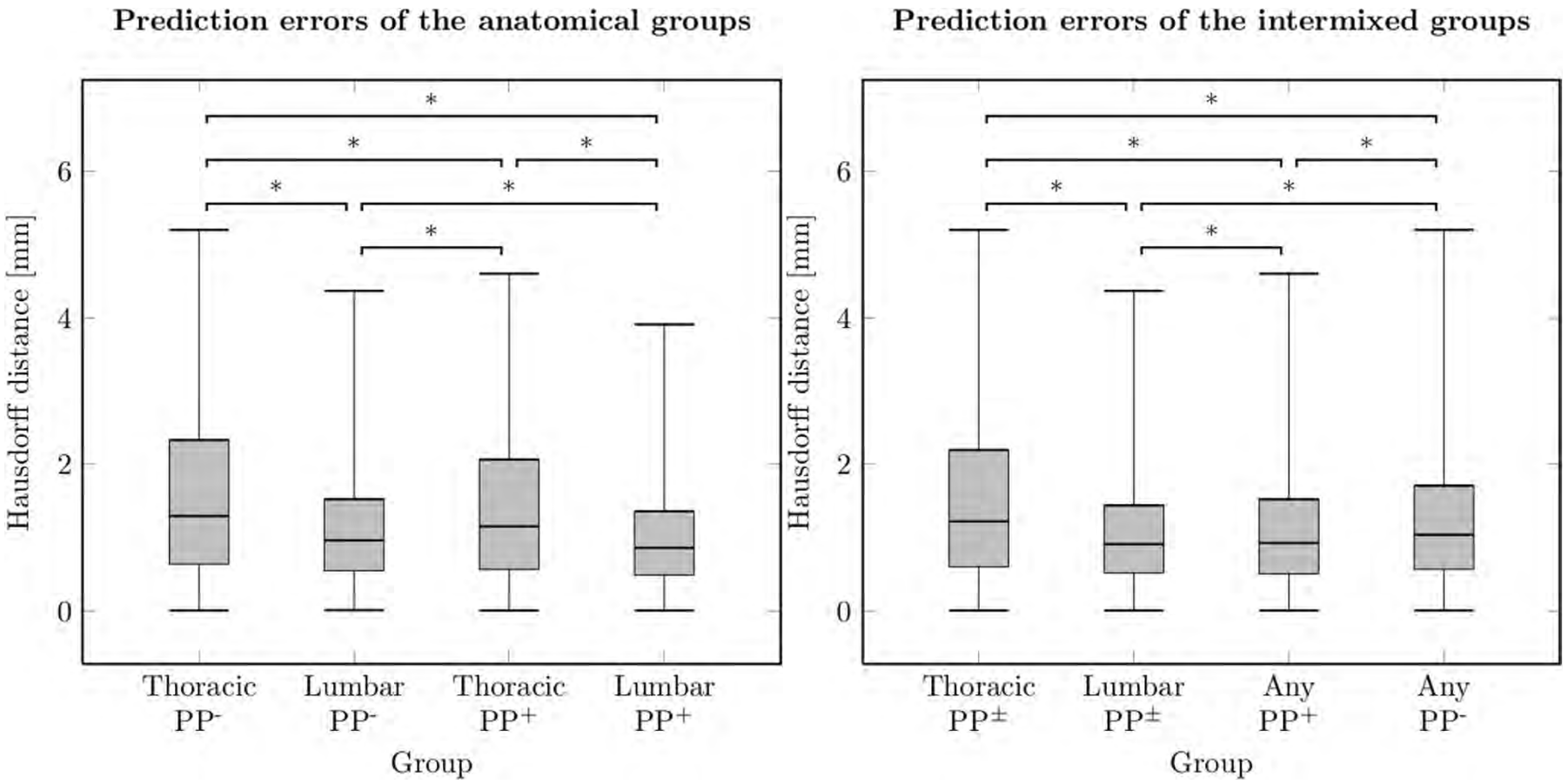


Results − Error Quantification (Hausdorff Distance)

$\mathrm{RMS}(e_d) = 1.6778\ \mathrm{mm}$

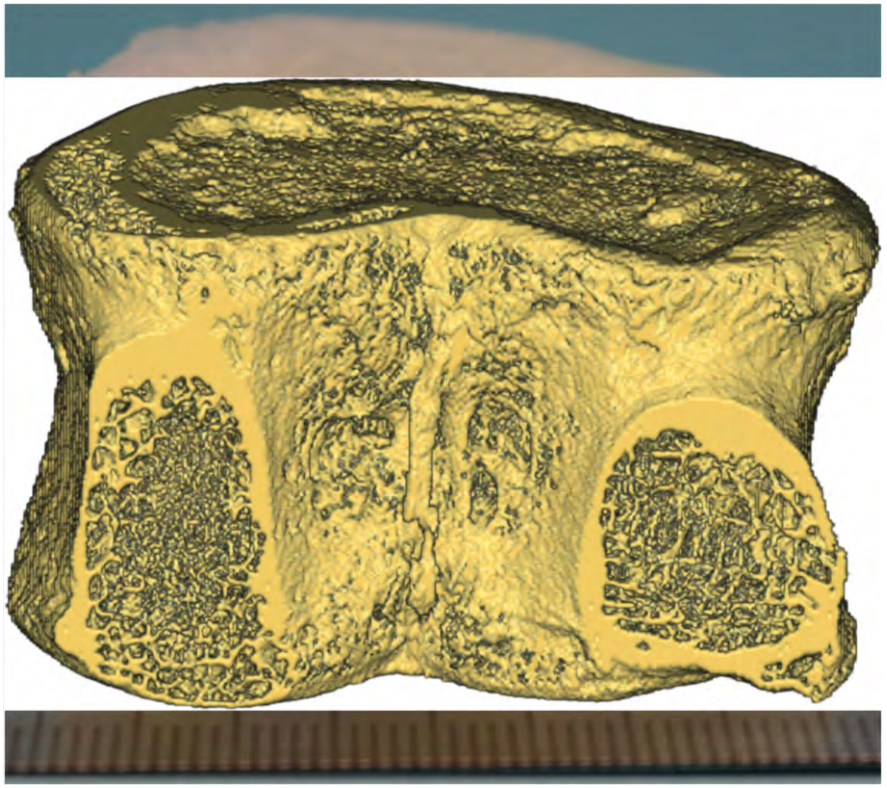


Results – Significance of Error Improvement



Experimental Validation – Surrogate-Bones

- Specific aims
  - Evaluation of bone cement and marrow substitutes
  - Design of an experimental setup to perform injections under multi-planar fluoroscopy
  - 3D-Reconstruction of the spreading pattern
  - Quantitative comparison to computational flow simulations
- Vertebral body surrogates fabricated by 3D printing technologies[1] (EUR 640.-/sample)

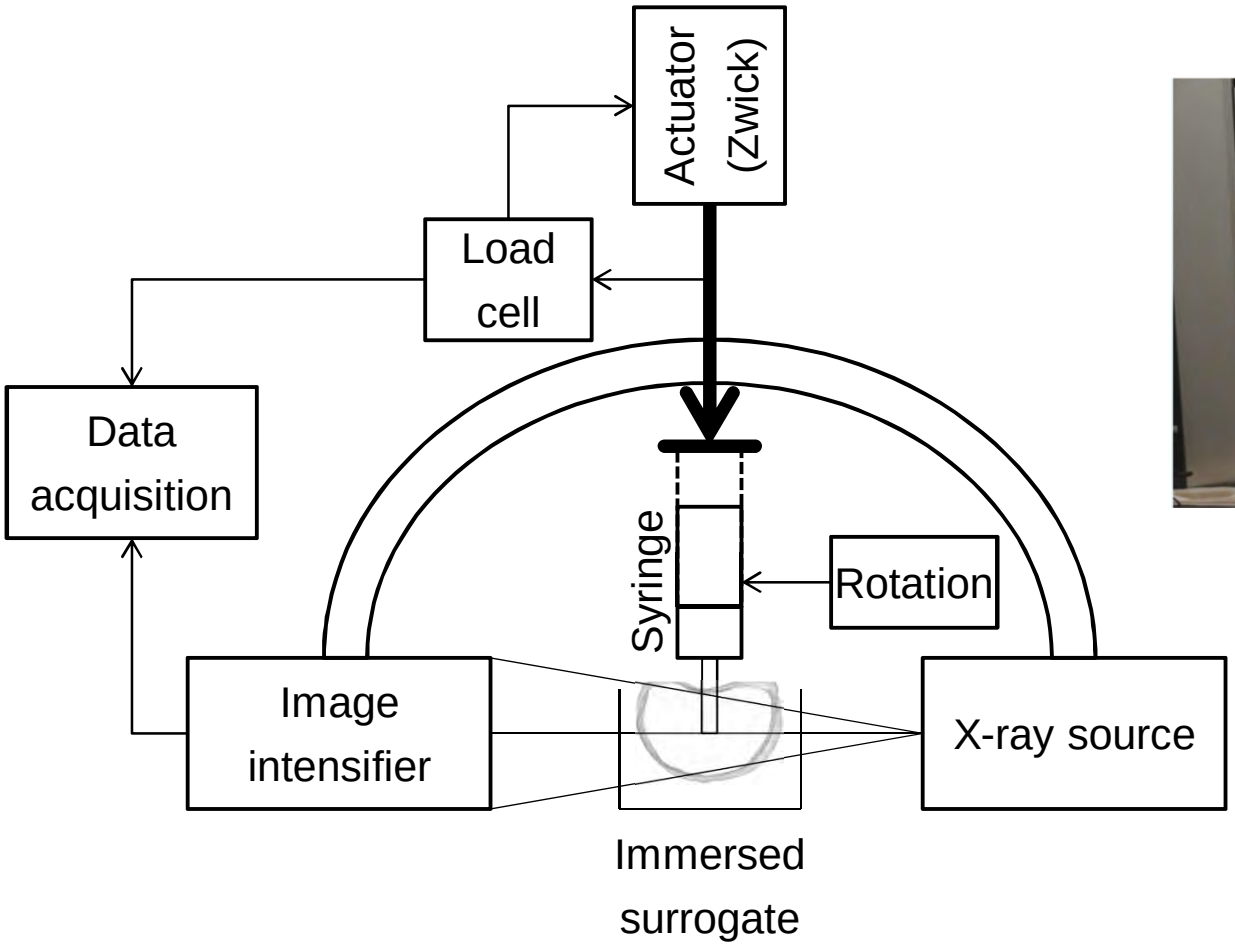


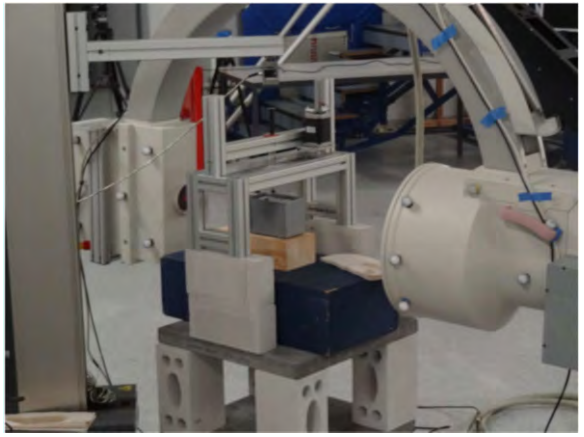
Surrogate bone

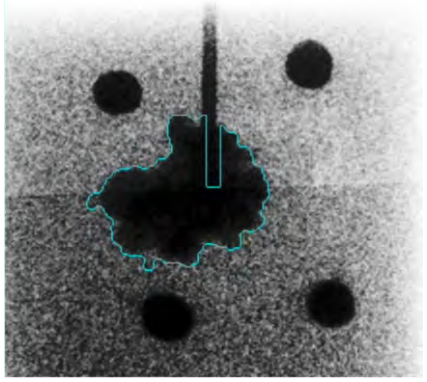
µCT scan

[1]Inspire, www.inspire.ethz.ch

Experimental Setup



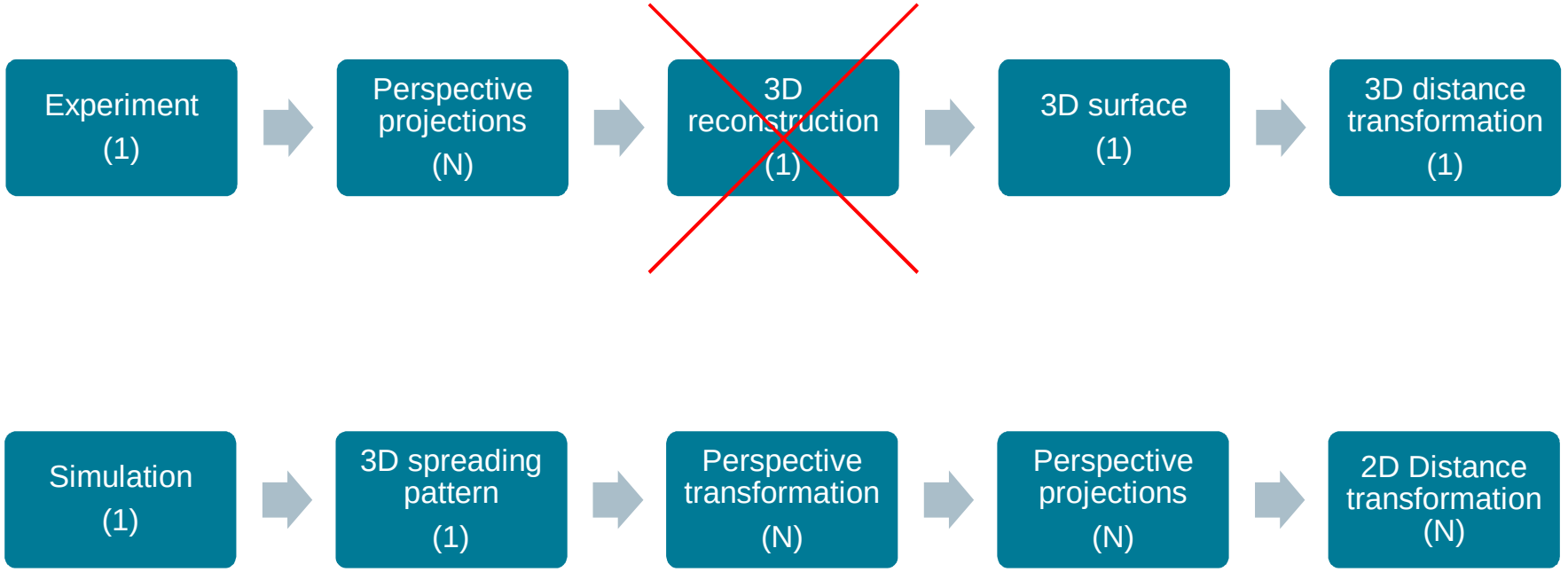




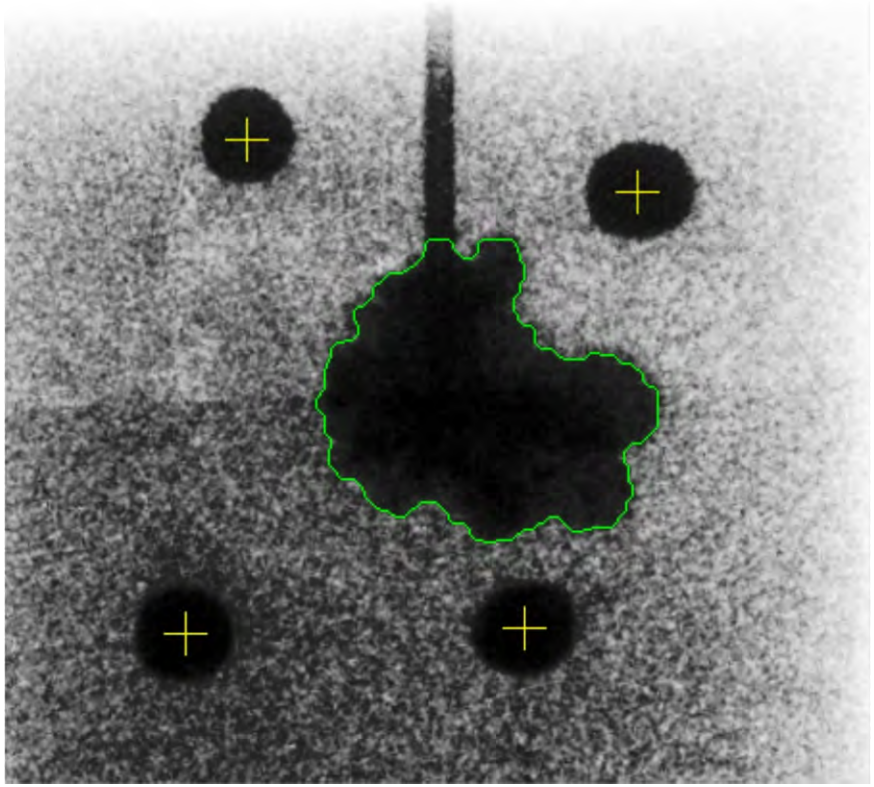
Flow Simulations

- Tetrahedral mesh derived from downsampled µCT stack (approx. 100K elements)
- BVTV, DA, Tb.Sp and Tb.Th
  - determined for cubic regions
  - evaluated at nodes of FE mesh
  - Interpolated to integration points
- Simulations performed
  - 2 specimens (#4022 L1 and #4024 L1)
  - 4 permeability models
    - BVTV ("BVTV")
    - BVTV + DA ("DA")
    - BVTV + DA + Tb.Sp ("Tb.Sp")
    - BVTV + DA + Tb.Sp + Tb.Th ("Tb.Th")

3D Reconstruction vs. Fluoroscopy Simulation



Perspective Registration

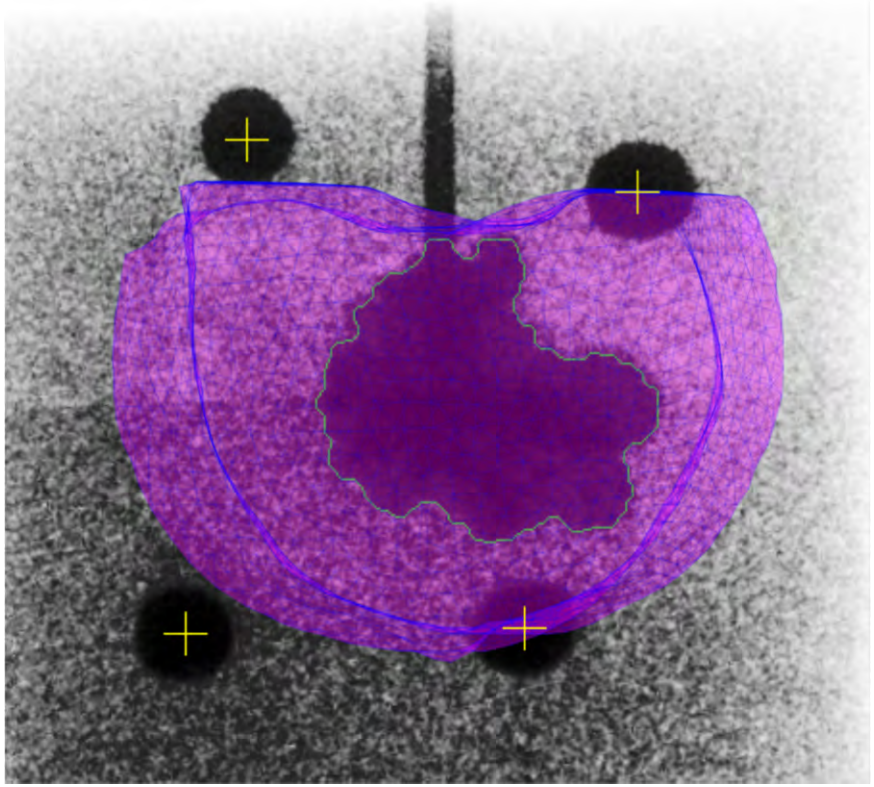


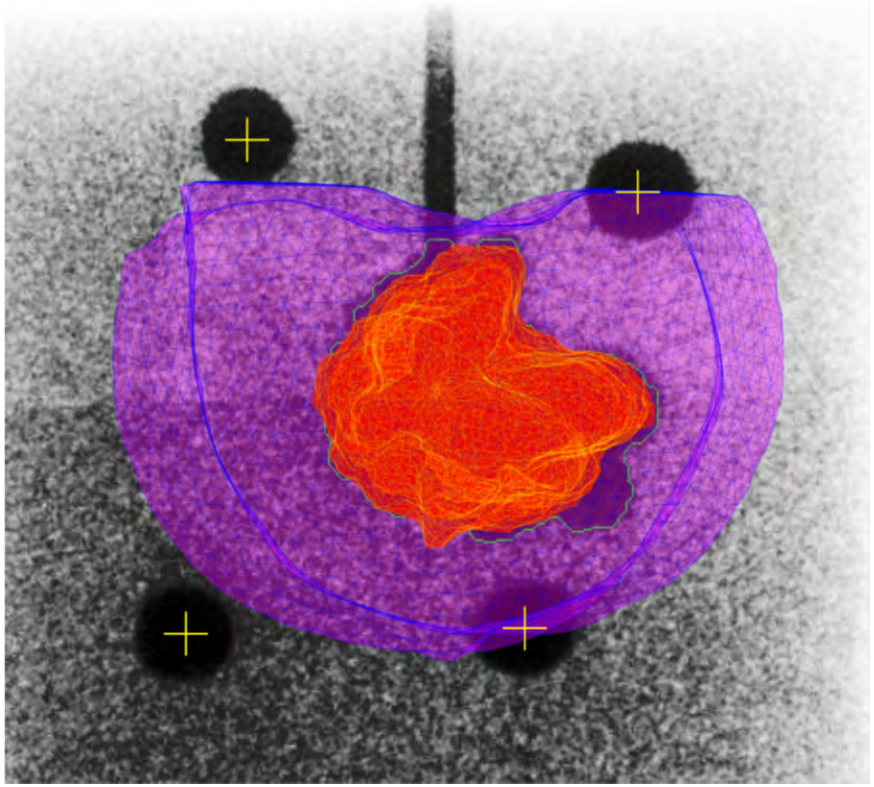
- Transformation model:

$$\underbrace{\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}}_{\underline{x}'} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\boldsymbol{T}} \cdot \underbrace{\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}}_{\underline{x}}$$

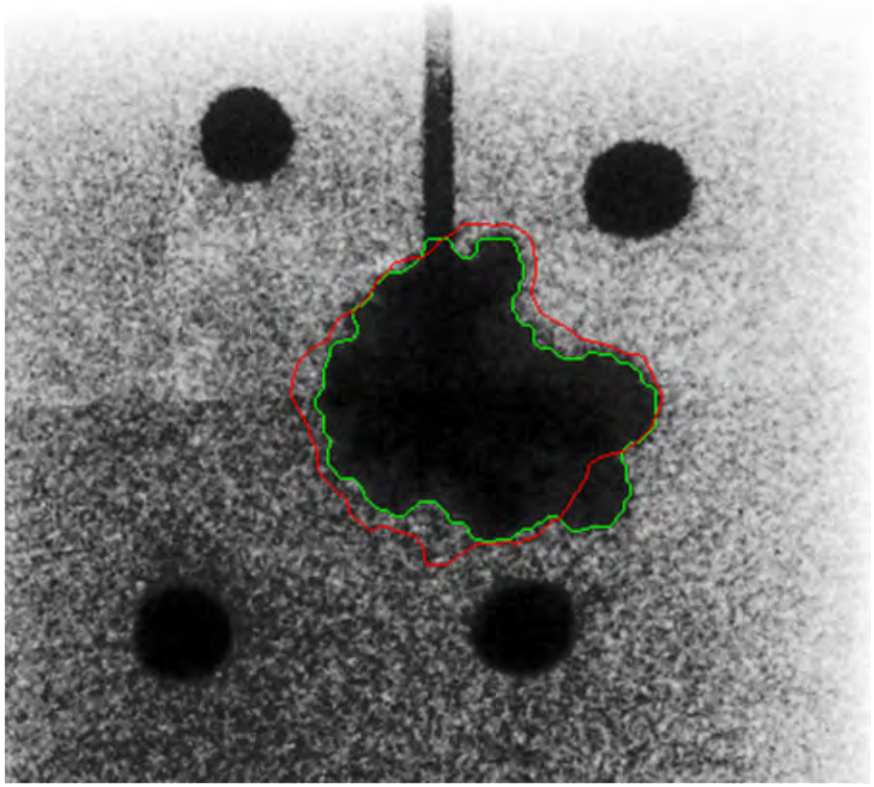
- $\boldsymbol{T}$: Transformation matrix
- $\underline{x}$: 3D **center of mass points** in simulation domain
- $\underline{x}'$: 2D **center of mass points** in experimental domain
- → Solve for $\boldsymbol{T}$

Perspective Projections



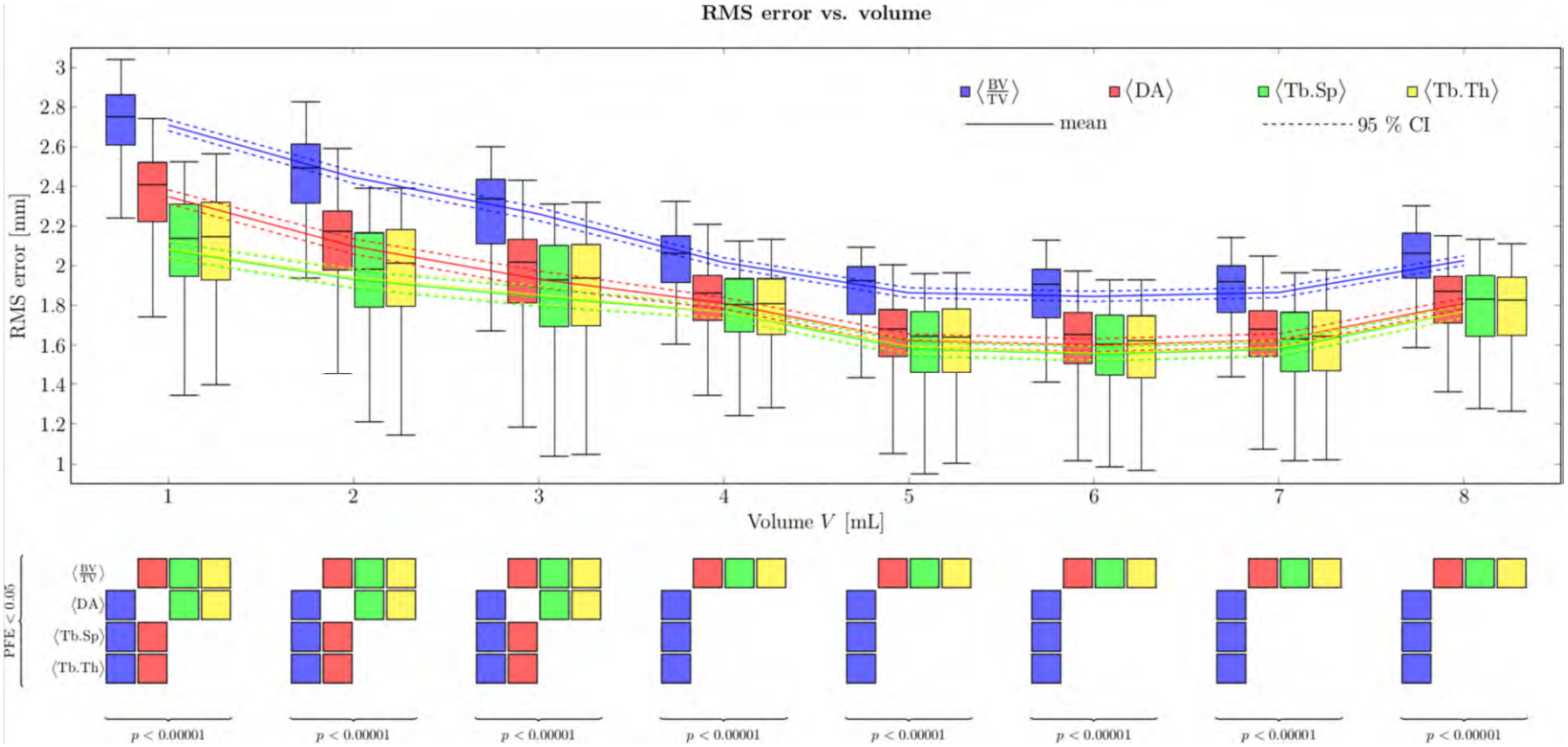


Error Estimation



- Distance transformation:
  $$\varepsilon = \mathrm{DT}(\mathrm{red}, \mathrm{green})$$
- DT: Distance transformation
- **Red**: Outline of **predicted** cement bolus projection
- **Green**: Outline of **estimated** cement bolus projection

RMS Error vs. Morphological Information



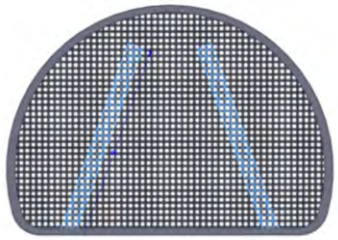
Discussion / Conclusions

- 3D reconstruction from fluoroscopy data delicate task
- Validation indicates RMS prediction error of 2…3 mm
- ≈2x RMS error of cadaveric study
  - registration error
  - micro-architectural mismatch of the bone surrogates
  - non-Newtonian fluid models
- Error depends on
  - injection volume (smaller if cement boundary is far away from inlet and outlet regions)
  - permeability model
- **Costs**

Outlook (I) – 3D Bone Surrogate Models

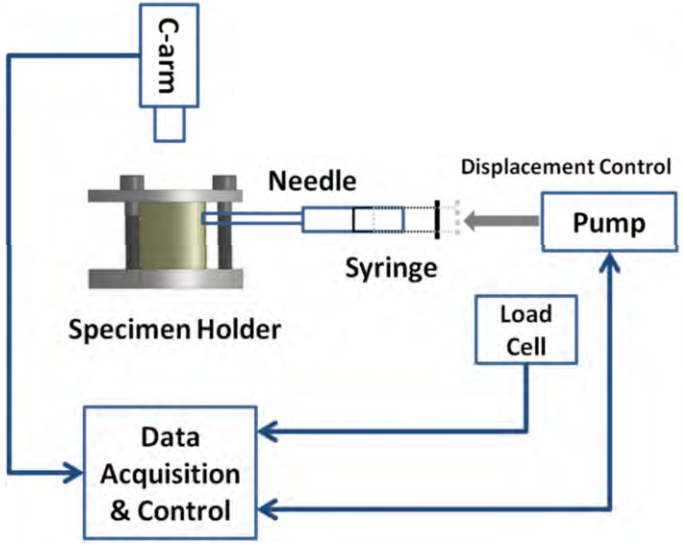
Anterior

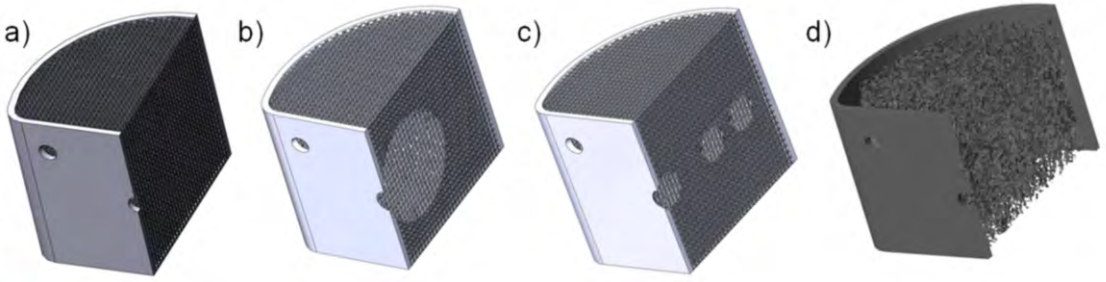
Posterior

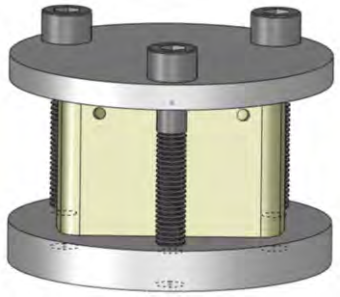


Superior

A. Francis / University of Leeds







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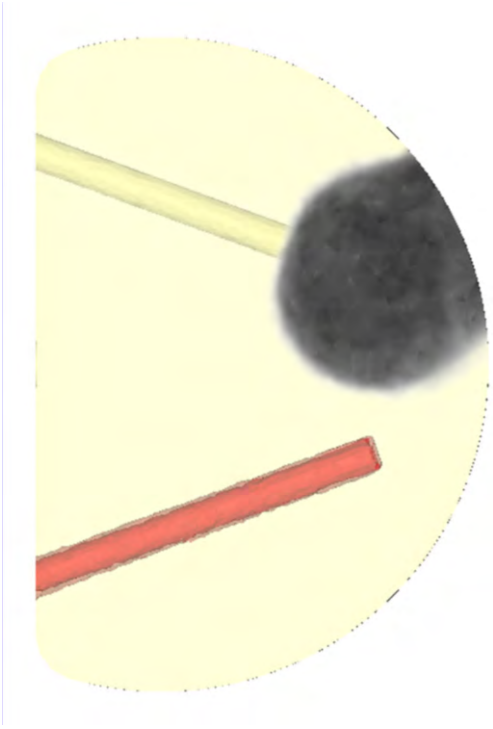
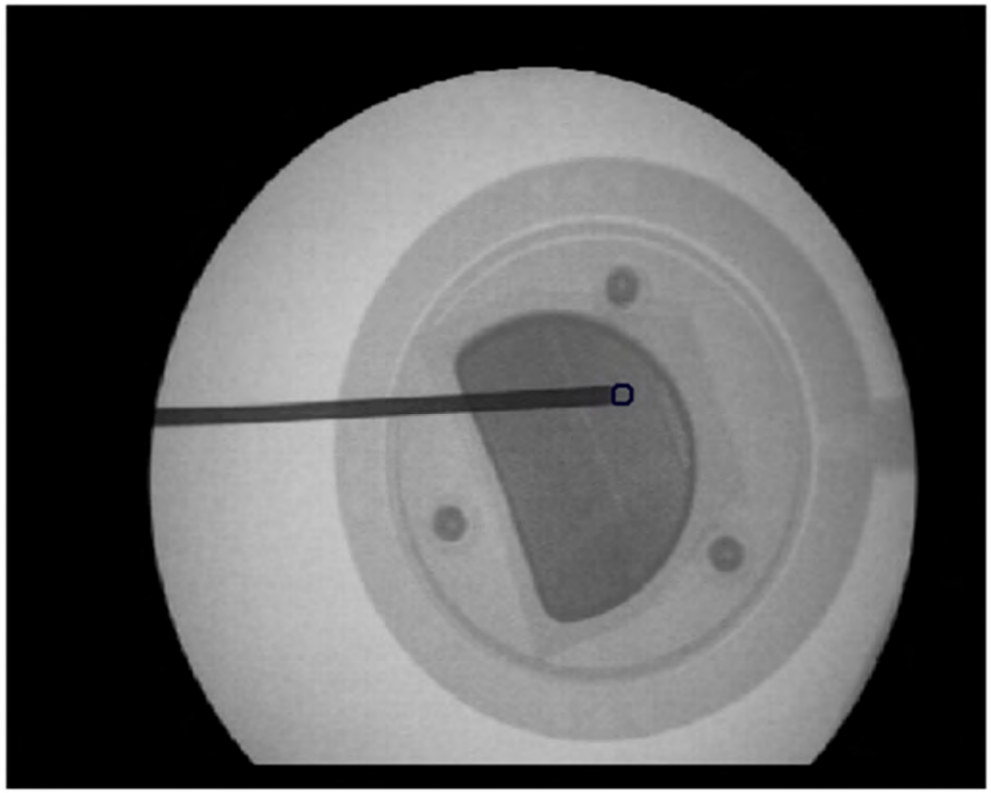
Outlook (II) – 3D Bone Surrogate Models

Simulation



Experiment



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Acknowledgements

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SEVENTH FRAMEWORK
PROGRAMME