Quantifying model quality using measured strain fields

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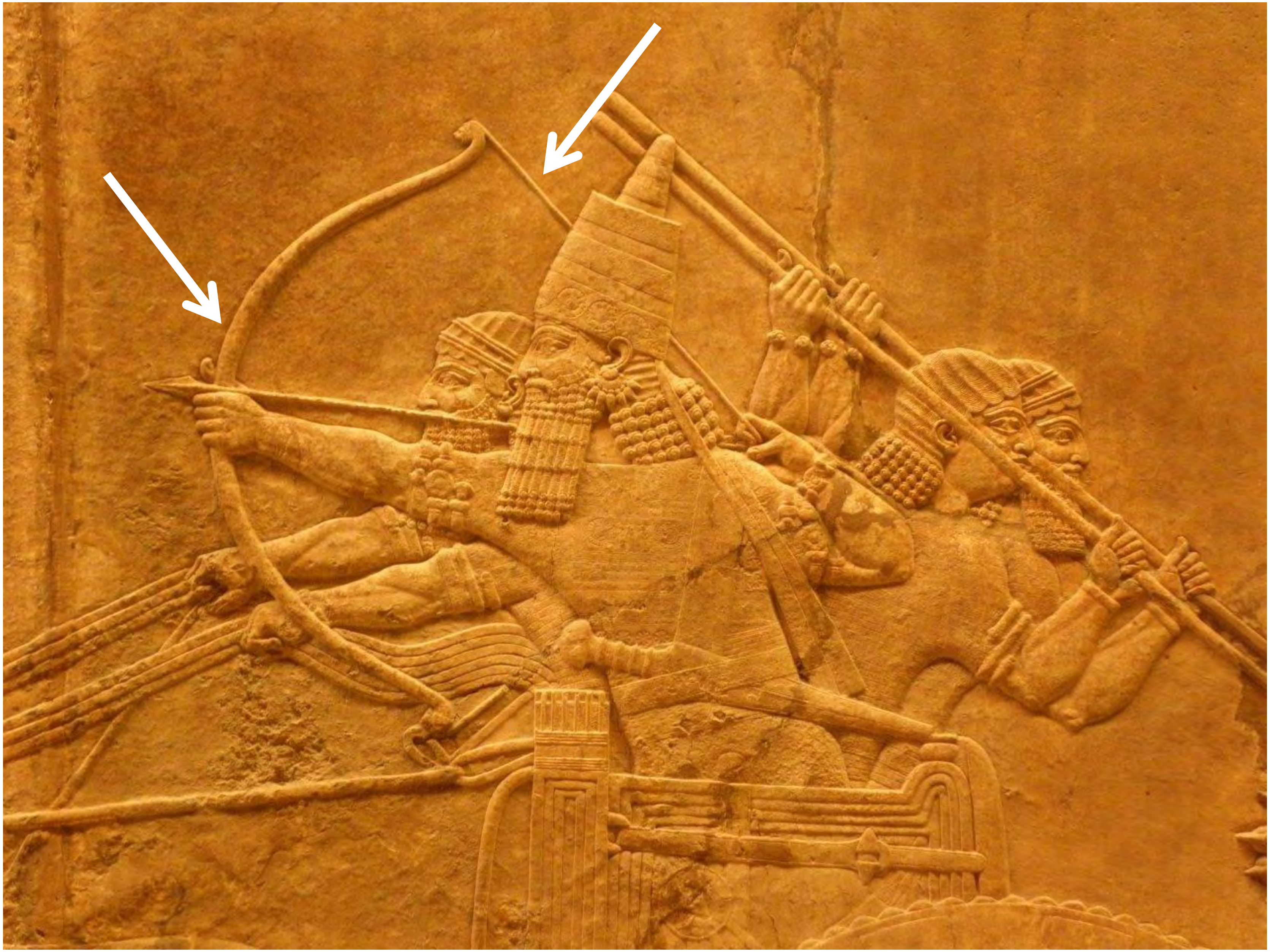

Bank Stress-Testing, Analysis, and Valuation - London

14–15 October 2013
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Summary

This is a highly practical and interactive course covering bank stress-testing, analysis, and valuation. T
begins with foundation work reviewing the basic bank business model and covering bank accounting a
regulation under Basel 3.

Exercises and workshops emphasize the role of stress-testing banks' capital and liquidity, dividing bank
that are more susceptible to bank failure and those that are likely to survive. The appropriate selection
gone-concern and going-concern valuation methods is then deployed to value banks' equity and credit

Continuing Education Information

As a provider-level participant in the Approved-Provider Program, London Financial Studies has deter
this program is eligible for continuing education credit. Pending provider confirmation of attendance, C
participation in this program will be recorded in individual CFA Institute members' CE record.

Topics

Risk Management

Content

- Damage and failure criteria
- Strain field measurement in 1D, 2D, and 3D
- Data compatibility of data-rich maps
- Image decomposition methods
- Quantification of model quality
- Conclusions

Stress, strain and damage

- **Many fracture/yield/plasticity criteria are based on stress (or strain) values**
  - von Mises, Tresca, Puck, **Logan-Hosford**

$$F\left|\sigma_2-\sigma_3\right|^n+G\left|\sigma_3-\sigma_1\right|^n+H\left|\sigma_1-\sigma_2\right|^n={\sigma_y}^n$$

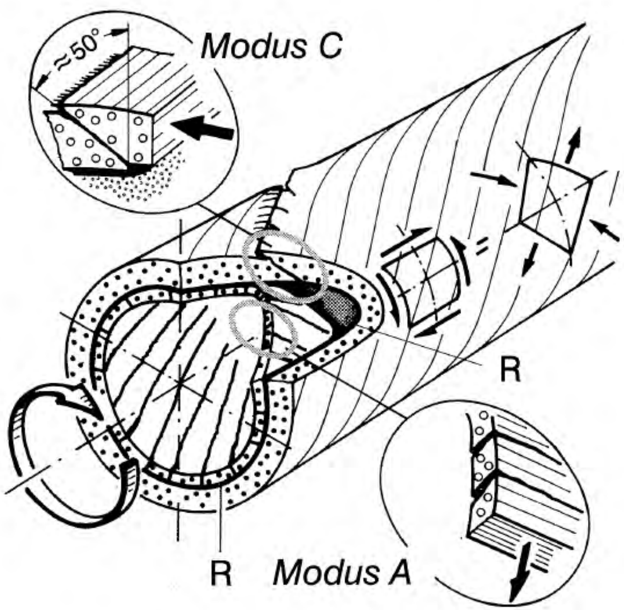
- **Local stress values are difficult to obtain for a component under service load**

- **Use defined stress cases in model systems first**

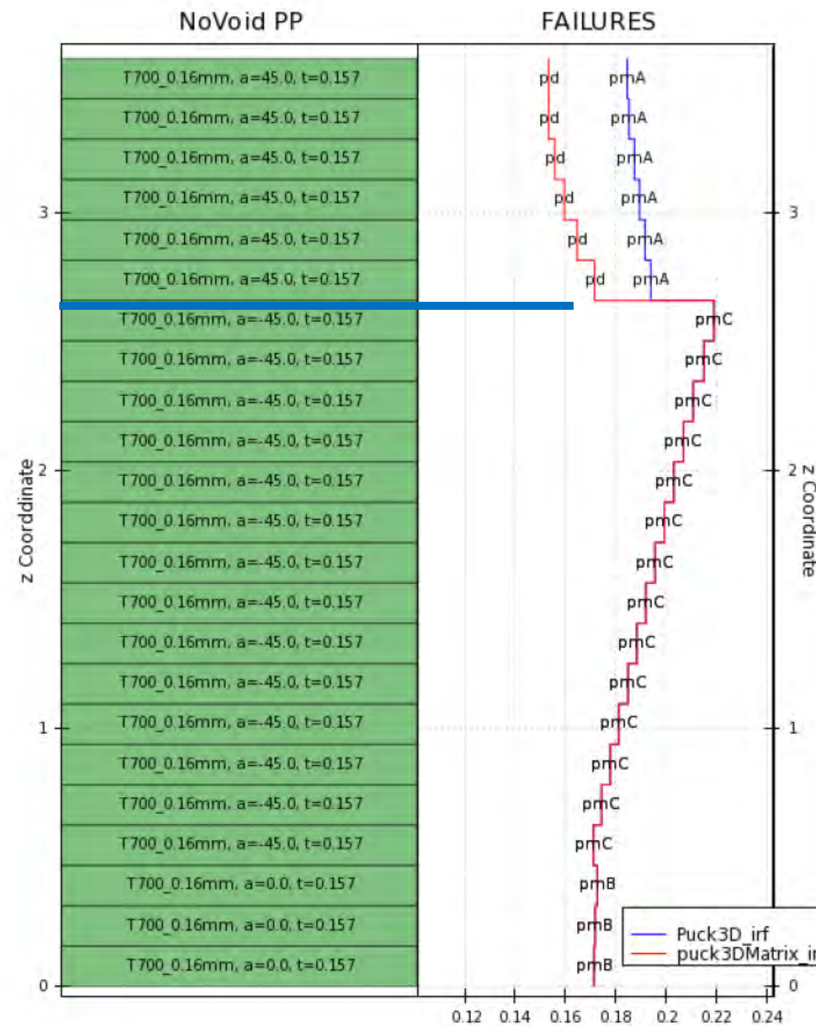
Case study: Torsion tube

Initial predicted failure/strength without voids:

- Critical failure mode: Matrix failure due to compression (pmC)
- Critical interface: -45/45
- Limit load (first ply failure): 3'700 Nm

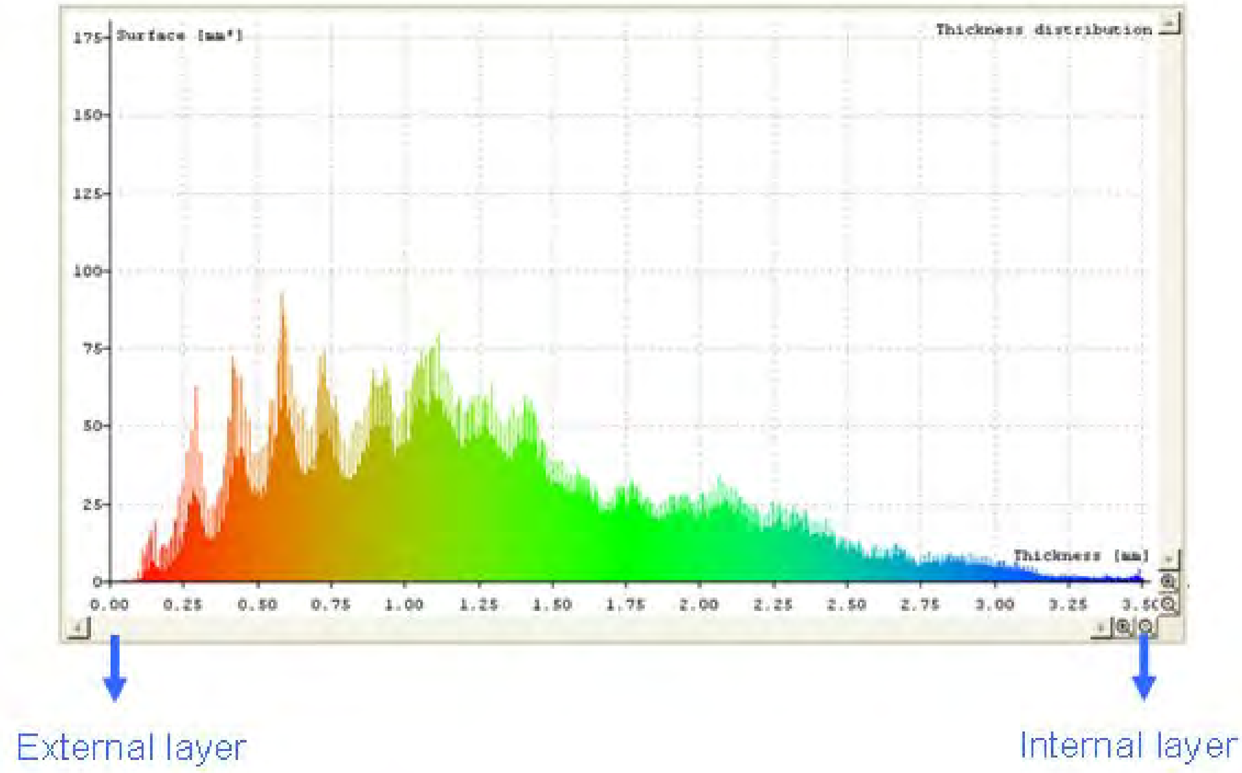


A. Puck, H. Schürmann / Composites Science and Technology 62 (2002) 1633–1662



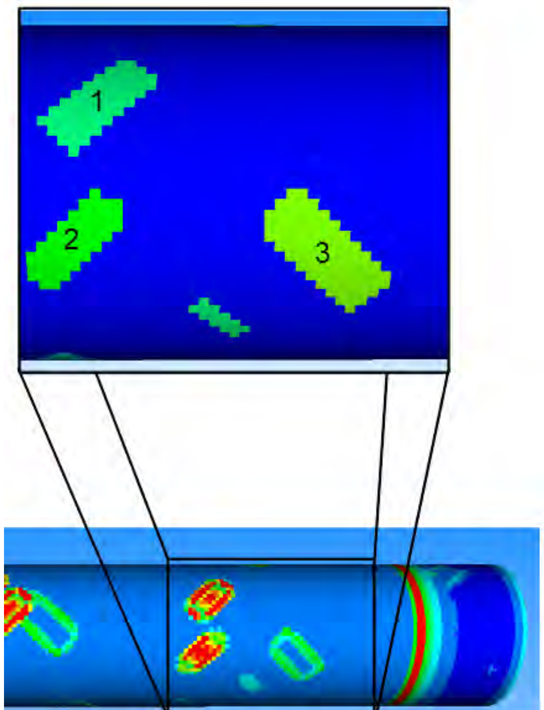
Volume data

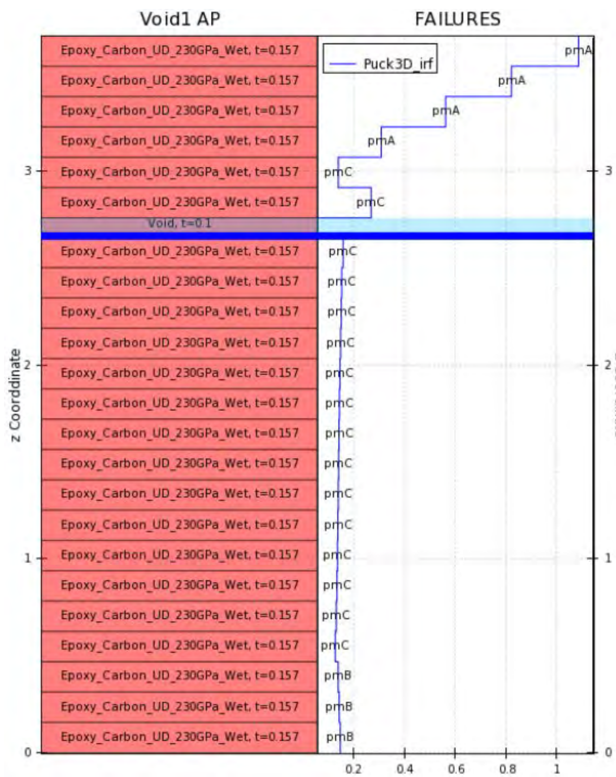
- **distribution of voids from x-ray CT**

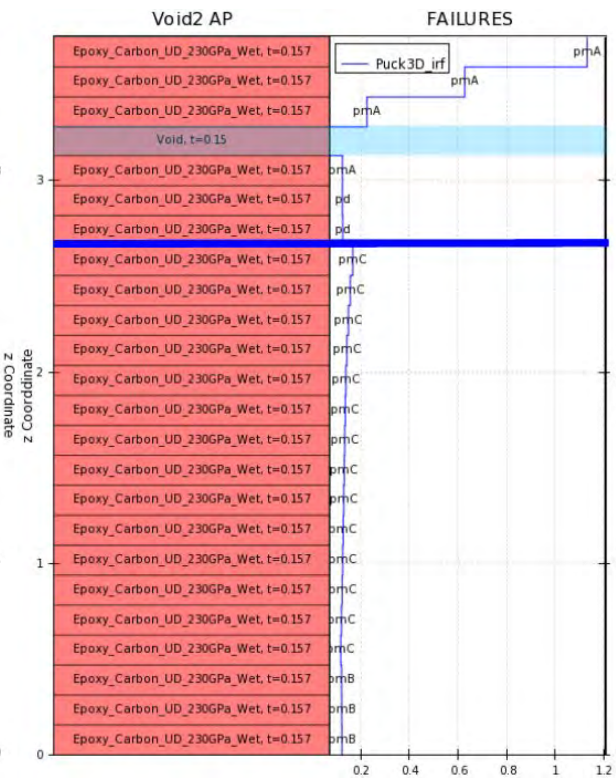


E. Hack, M. Feligiotti, R.K. Fruehmann, and J.M. Dulieu-Barton, *Failure and damage in CFRP torsion tubes*, Photomechanics, Montpellier, 27-29 May 2013

3D-model for damage predicition

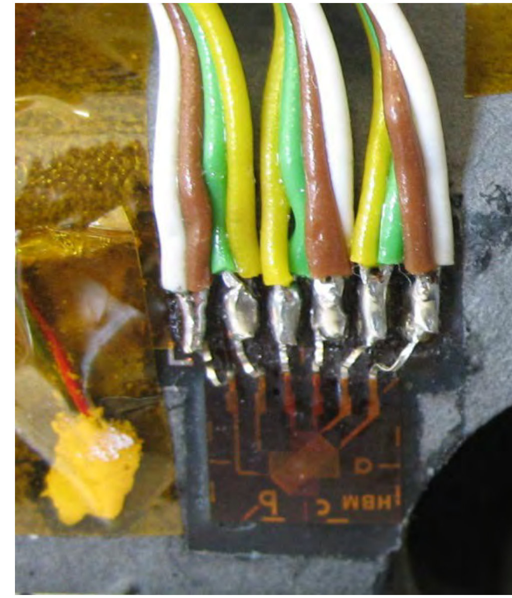




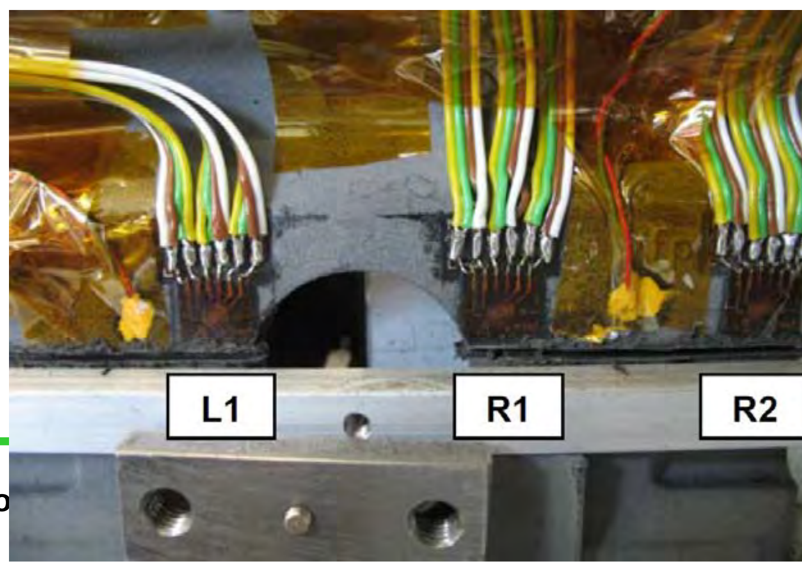


Case study: FRP injection moulded component

Specimen with direction of cracks induced by residual stress



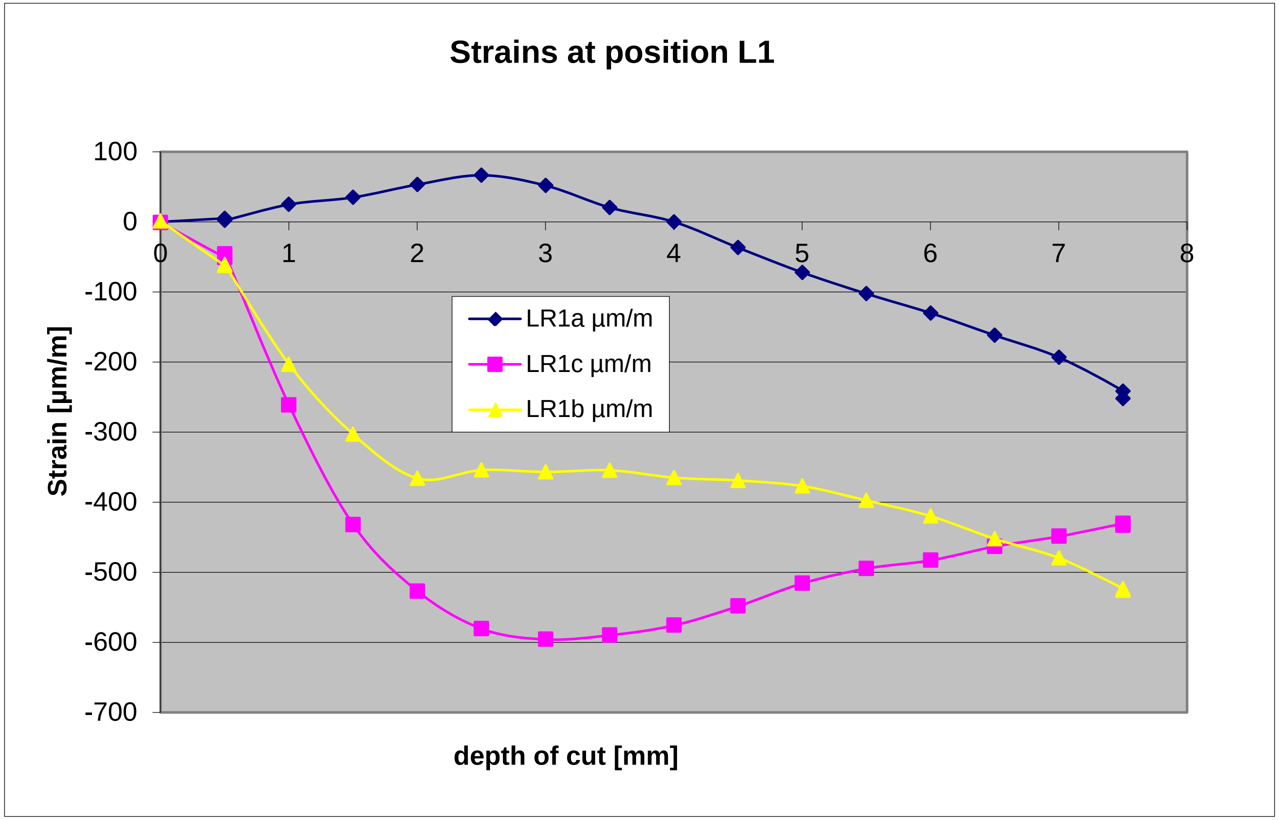
Mount for saw notch method

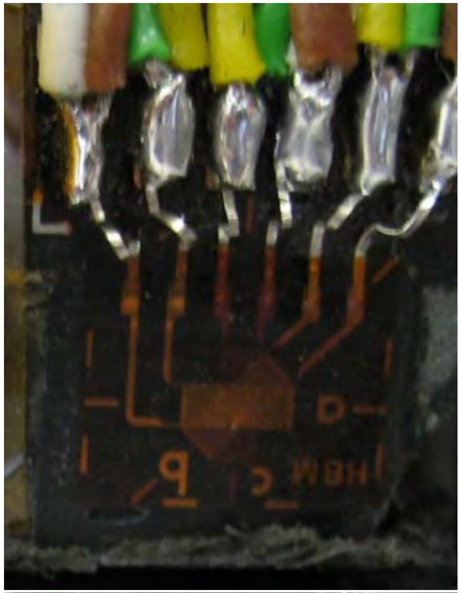


Local measurement of surface strains with RSG-Rosette

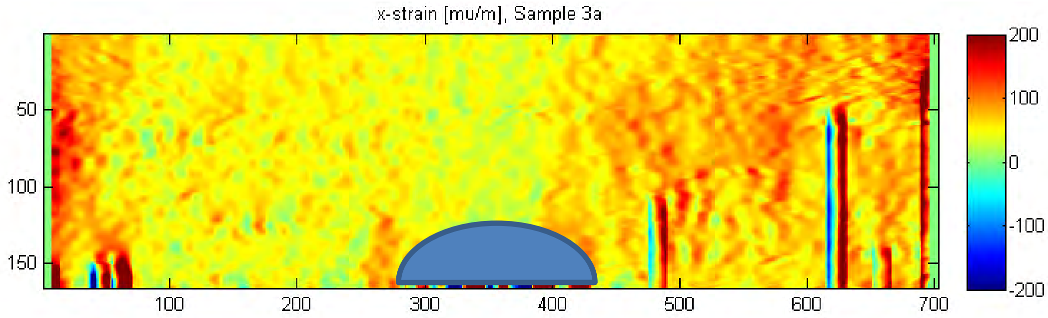
RSG: local strain data

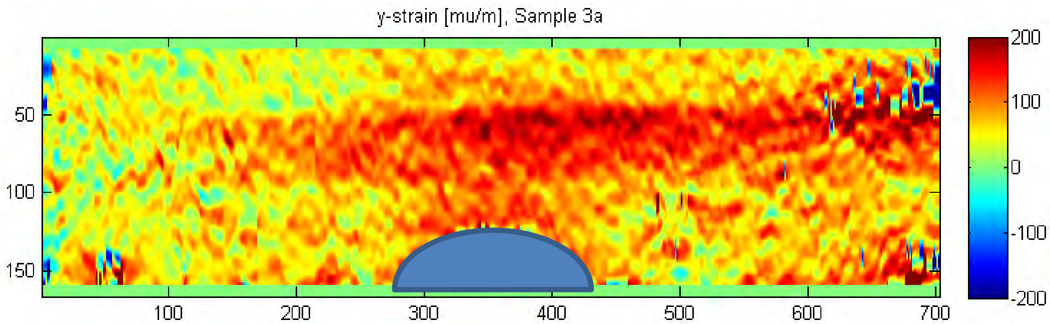
- **Measured at increasing notch depth**
  - after cooling down (thermocouple)

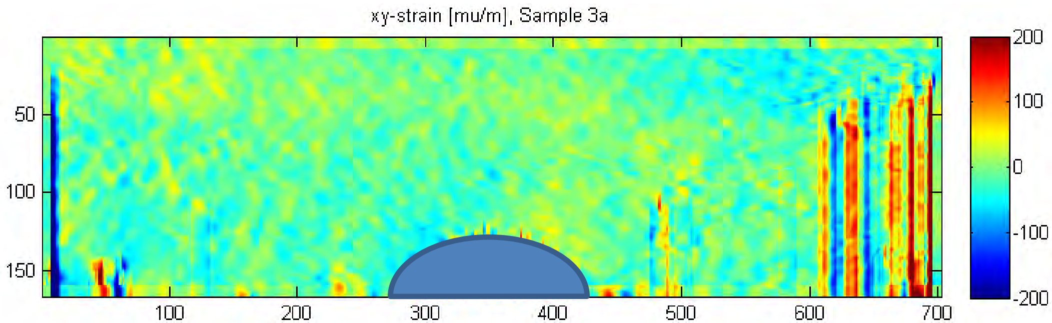




DSPI: global strain data

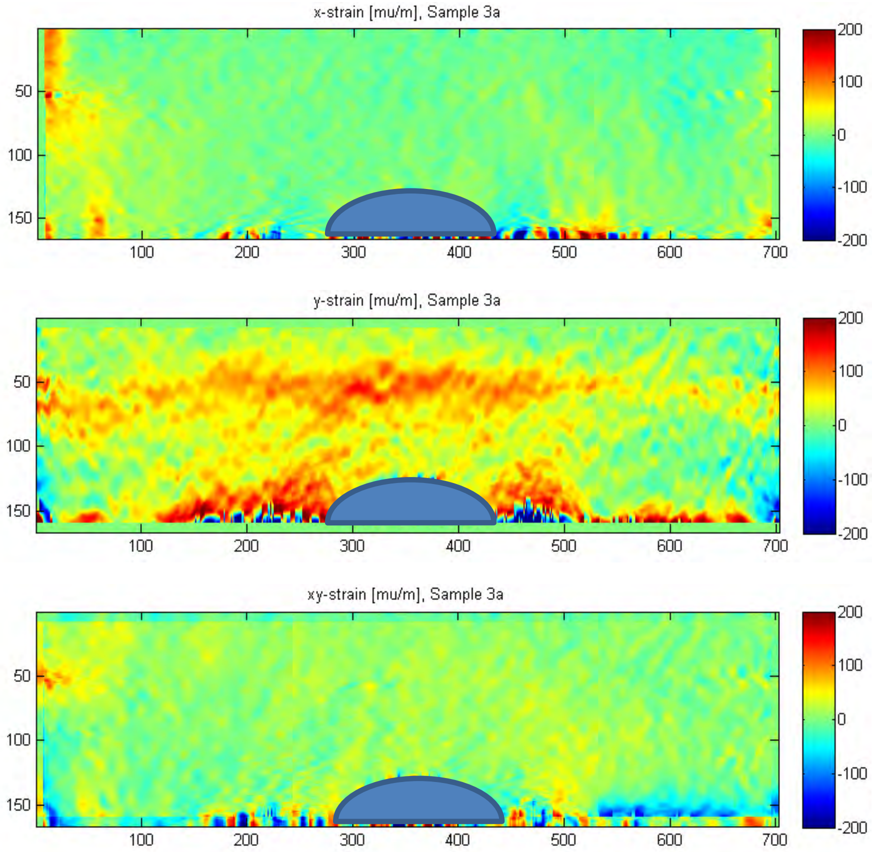


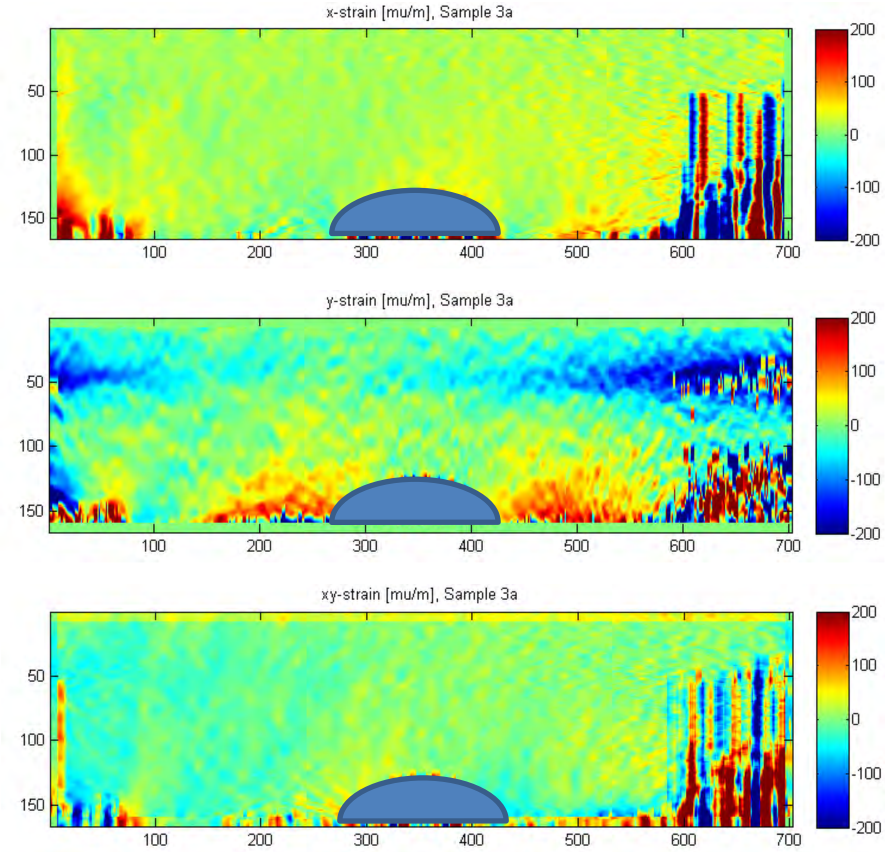




Strain values for saw notch
0.0 – 0.5 mm

DSPI: strain reversal





Strain values for saw notch
0.5 – 1.0 mm

Strain values for saw notch
1.0 – 1.5 mm

Surface strain

- Since x-ray CT cannot be used outside the lab volume data and volume strain are not accessible
- Yet measurement of surface strains is feasible, *in situ* and under defined load (stress)
- Some 3D information can be obtained destructively
- Commonly therefore, surface strain values are compared with numerical model data

Modeling quality

- **Numerical 3D-models calculate strain/stress fields and explain failure load and mode**
- **What are model quality issues?**
  - Appropriate physics
  - Code verification
  - Appropriate meshing
  - Appropriate boundary conditions
  - Convergence of solution
  - **Robustness of solution**
  - **Correctness of solution**

PREFACE

THANKS to the labors of Kirchhoff, Kelvin, Huxley, and others, there is now a widespread opinion that any physical phenomenon is "explained" only when some one has devised a dynamical model which will duplicate the phenomenon. The completeness of the explanation is to be measured by the completeness with which the model will duplicate the phenomenon.

Henry Crew (Editor), *The wave theory of light: memoirs by Huygens, Young and Fresnel*, American Book Company, New York 1900.

- **This statement refers to a physical theory, but…**
- **could a similar statement hold true for numerical models?**

Faithful representation

- **«Devise a dynamical model»**
  - set-up a numerical model
- **«Duplicate the phenomenon»**
  - explain experimental outcome
- **«Completeness of explanation»**
  - quantify deviations

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Faithfulness and validation

- **VIM definition (JCGM 200:2012)**

- **Validation** is defined as the provision of objective evidence that a given item fulfils specified requirements adequate for an intended use.

- Intended use: **Fitness for purpose**

«Duplicate the phenomenon»:
Comparison of model and measurement data

- **General concept** in analogy to **metrological compatibility of measurement results**
- **VIM 2012:**
  "property of a set of **measurement results** for a specified **measurand**, such that the absolute value of the difference of any pair of **measured quantity values** from two different measurement results is smaller than some chosen multiple of the **standard measurement uncertainty** of that difference"

$$\left|\varepsilon_{1n} - \varepsilon_{2m}\right| \leq \kappa \times u(d) \quad \forall n, m$$

Single measurement value criterion!

Point-wise comparison of data fields

$$d(\mathbf{x}_M) = \varepsilon_{opt}(\mathbf{x}_M) - \varepsilon_{FEM}(\mathbf{x}_M)$$

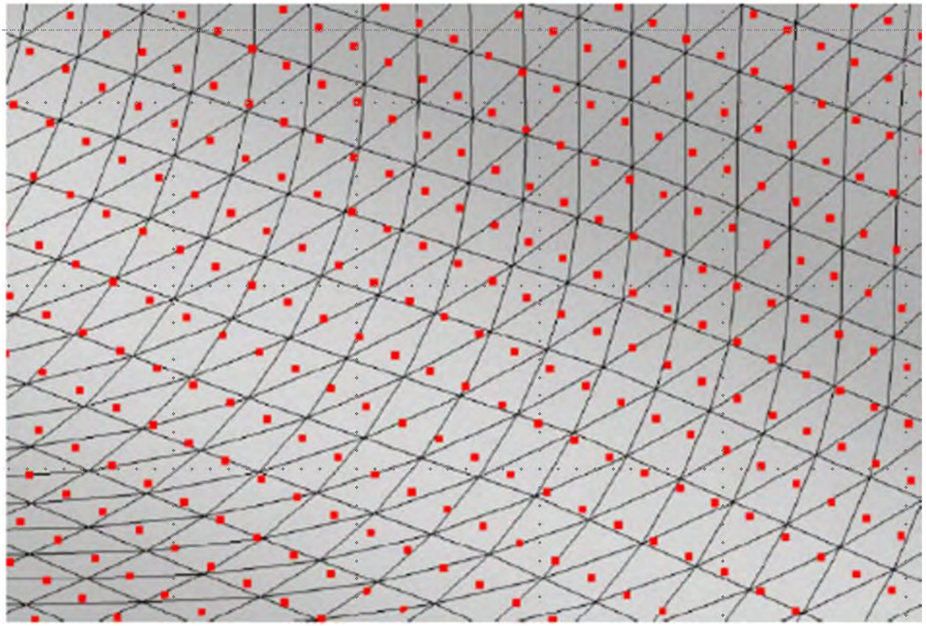
- **Data sets are expressed as *N*-dimensional vectors**
- **An overall quantitative quality criterion is a must**
  - e.g. when different FEA results based on different models or different parameter values are available.
- **The following slide shows an indicative list of possible quality criteria (“cost functions”, “distance measures”).**
  - All summations are weighted summations, but we suppress the weighting factors for clarity.

Quality criteria for a set of points

- max. allowable deviation: $$Max = \max\left(\left|d(\mathbf{x}_M)\right| M = 1 \ldots N\right)$$
- **rms criterion:** $$rms = \frac{1}{N}\sqrt{\sum_{M=1}^{N} d^2(\mathbf{x}_M)}$$
- Modal Assurance Criterion: $$MAC = \frac{\left|\sum_{M=1}^{N} \varepsilon_{opt}(\mathbf{x}_M) \cdot \varepsilon_{FEM}(\mathbf{x}_M)\right|^2}{\sum_{M=1}^{N} \left|\varepsilon_{opt}(\mathbf{x}_M)\right|^2 \sum_{M=1}^{N} \left|\varepsilon_{FEM}(\mathbf{x}_M)\right|^2}$$
- Normalised cross correlation: $$r = \frac{N\sum_{M=1}^{N} \varepsilon_{opt}(\mathbf{x}_M) \cdot \varepsilon_{FEM}(\mathbf{x}_M) - \sum_{M=1}^{N} \varepsilon_{opt}(\mathbf{x}_M) \sum_{M=1}^{N} \varepsilon_{FEM}(\mathbf{x}_M)}{\sqrt{N\sum_{M=1}^{N} \left|\varepsilon_{opt}(\mathbf{x}_M)\right|^2 - \left(\sum_{M=1}^{N} \varepsilon_{opt}(\mathbf{x}_M)\right)^2} \sqrt{N\sum_{M=1}^{N} \left|\varepsilon_{FEM}(\mathbf{x}_M)\right|^2 - \left(\sum_{M=1}^{N} \varepsilon_{FEM}(\mathbf{x}_M)\right)^2}}$$

Point-wise comparison

- Comparison can be performed point by point and the criterion be applied
- Involves step of point matching and data interpolation



Measurement points (red) vs. FEA grid (lines)

Unequal number of points?

- **Point-to-point comparison requests** $N_{opt} = N_{FEM}$
  - Missing data points generated by intra-/extrapolation
- **Is there a different way of comparing unequal sets of data?**
- **Idea: Parameterize field of data points and compare the reduced data**

$$\varepsilon_{opt}(x,y) = \sum_{nm} a_{nm} f_n(x) f_m(y)$$

$$\varepsilon_{FEM}(x,y) = \sum_{nm} b_{nm} f_n(x) f_m(y)$$

$$\varepsilon_{opt}(x,y) - \varepsilon_{FEM}(x,y) = \sum_{nm} (a_{nm} - b_{nm}) f_n(x) f_m(y)$$

Orthogonal basis systems

- **For circular domain : Zernike polynomials**
- **For rectangular domain:**
  - Fourier components
  - Orthonormal polynomials
  - Discrete version
  - e.g. discrete Chebyshev polynomials

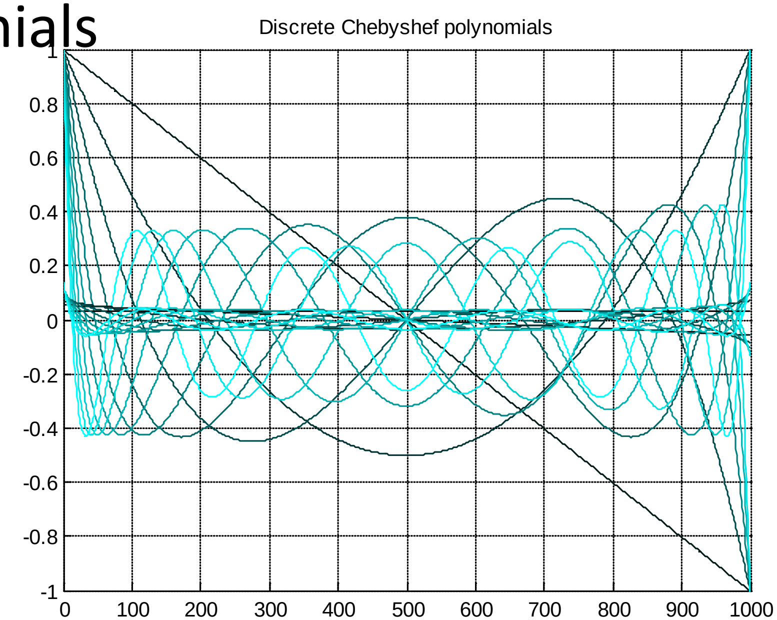
$$\int f_m(x) f_n(x) W(x)\, dx = \delta_{mn}\rho_n$$

$$\sum_{p=0}^{N-1} f_m(p) f_n(p) W(p) = \delta_{mn}\rho_n$$

$$T_0(p) = 1 \qquad p \in [0, N-1]$$

$$T_1(p) = 1 - \frac{2p}{N-1}$$

$$T_{n+2}(p) = \frac{1}{(n+1)(N-1-n)}\left[(2n+1)(N-1-2p)T_{n+1}(p) - n(n+N)T_n(p)\right]$$



Normalisation issue

- **The coefficients should not depend (explicitly) on number of points.**

$$\varepsilon(p) = \sum_{n=0}^{P} a_n f_n(p)$$

$$a_n = \sum_{p=0}^{N-1} \varepsilon(p) f_n(p)$$

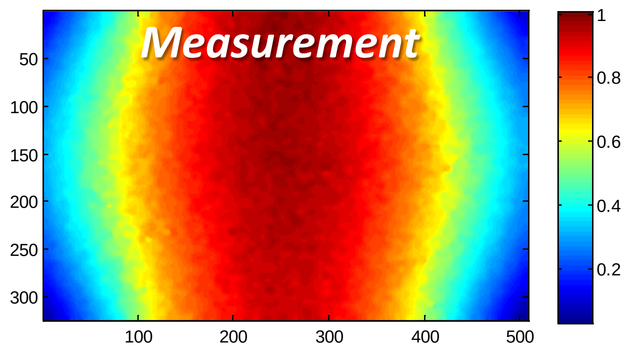
- **If ε(p) is a constant**

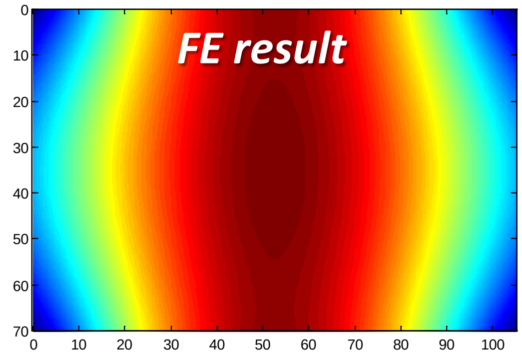
$$a_0 = \sum_{p=0}^{N-1} \varepsilon(p) f_0(p) \stackrel{!}{=} \frac{1}{N} \sum_{p=0}^{N-1} \varepsilon(p)$$

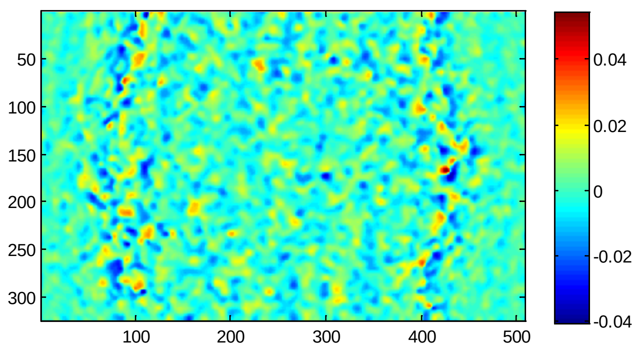
$$\sum_{p=0}^{N-1} f_m(p) f_n(p) = \delta_{mn} \rho_n$$

thus $f_0(p) = \frac{1}{N}$ **and** $\rho_0 = \frac{1}{N}$

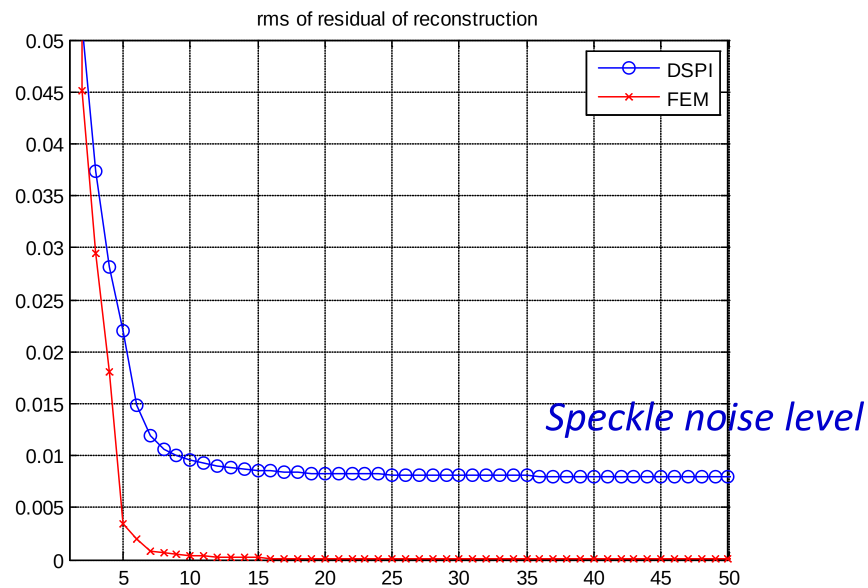
Image decomposition







Residuals for a 20 term reconstruction of DSPI map



Mean square residual after reconstruction

Methodology to compare experimental and simulation data

$d(\mathbf{x}_M) = \varepsilon_{opt}(\mathbf{x}_M) - \varepsilon_{FEM}(\mathbf{x}_M)$ is replaced by $d_{nm} = a_{nm} - b_{nm}$

M = O(10^6) points ***nm = O(10^3) coefficients***

- **The basis functions for *FEM* and *opt* are calculated on the respective domains**
- **The coefficients a_{nm} and b_{nm} have the same dimension as the measurand, viz. strain**
- **To display the differences it is best to sort the coefficients according to their value**

Compatibility of data

- **based on the *standard measurement uncertainty***

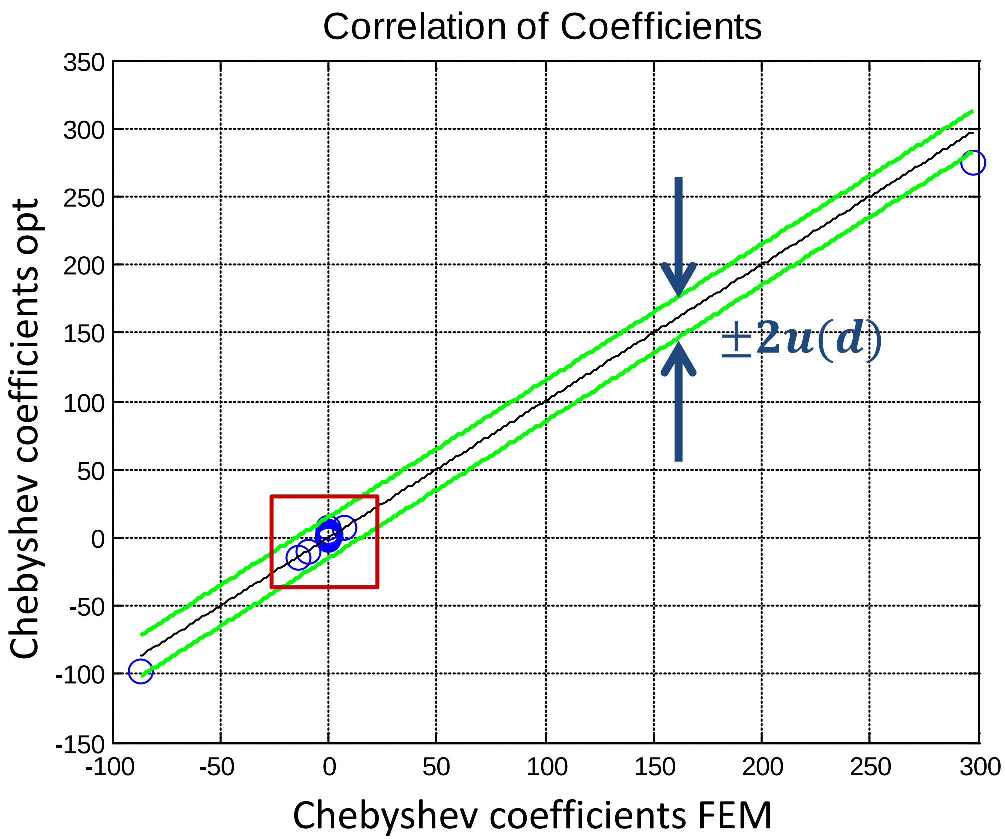
$$d_{nm} = a_{nm} - b_{nm}$$
$$u^2(d_{nm}) = u^2(a_{nm}) + u^2(b_{nm})$$

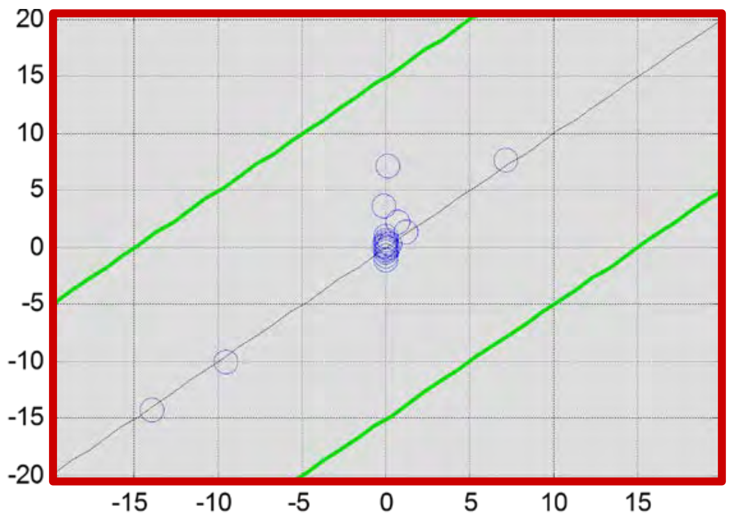
- **It can be shown that for orthonormal decomposition the uncertainties of the coefficients are equal for all *nm*.**

$$u^2(d) = u^2(a) + u^2(b) \qquad u^2(d) = u_{opt}^2 + u_{FEM}^2$$

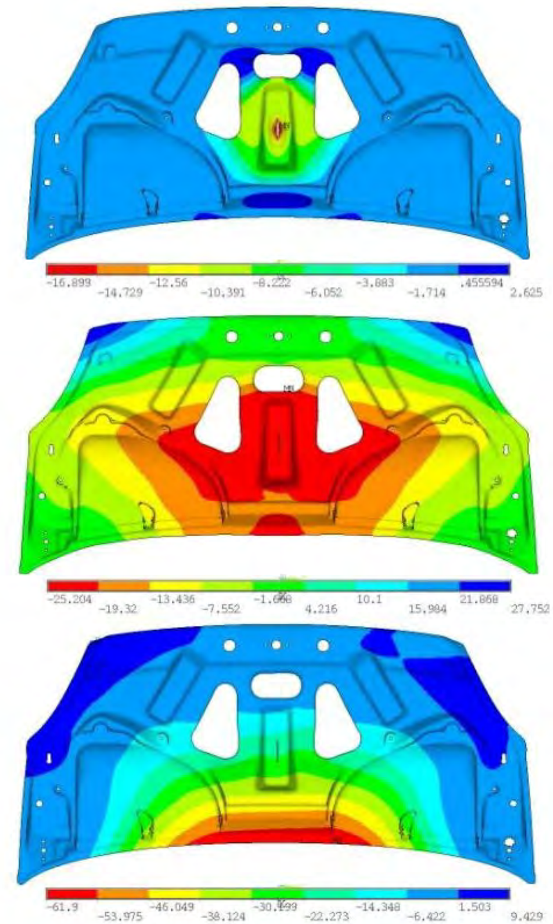
Comparison of coefficients

- **Orthonormal Chebyshev decomposition**





Prospect: non-rectangular domains



W. Wang and J.E. Mottershead, *Adaptive moment descriptors for full-field strain and displacement measurement* , J Strain Analysis 48 (2012) 16-35

Summary

- Motivated the use of full-field (surface) strain data based on failure criteria
- Viewed model validation in analogy to metrological compatibility of measurement results
- Suggested a method to compare full-field data sets by data reduction
- Suggested a criterion to quantify model quality for a validation test

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