Identification & modeling of complex nonlinear systems from uncertain/limited information

Prof. Dr. Eleni Chatzi
Institute of Structural Engineering, ETH Zürich

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Background & Motivation

Increased Complexity on the Material & Structural System level

Functionally Graded Materials

(source: MIT Media Lab)

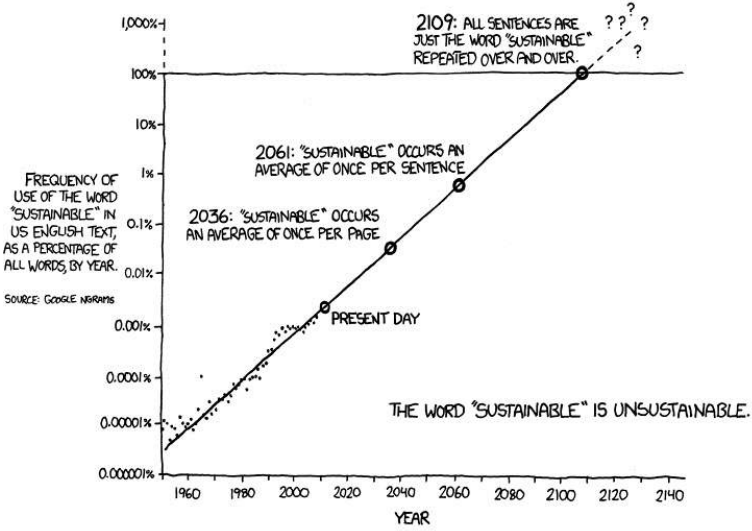
Pedestrian Vibrations
Millennium Bridge

Background & Motivation

Deteriorating Infrastructure

Ageing
Infrastructure demographic

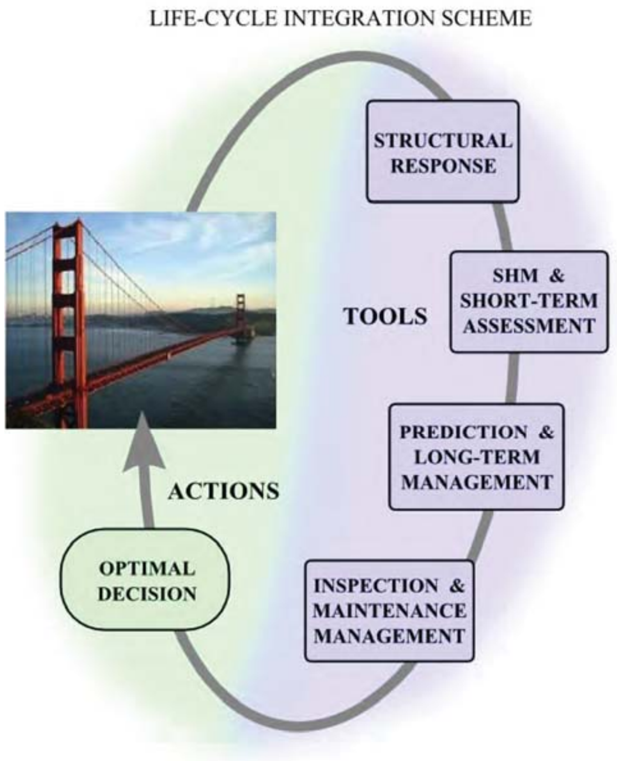
Quest for Sustainability

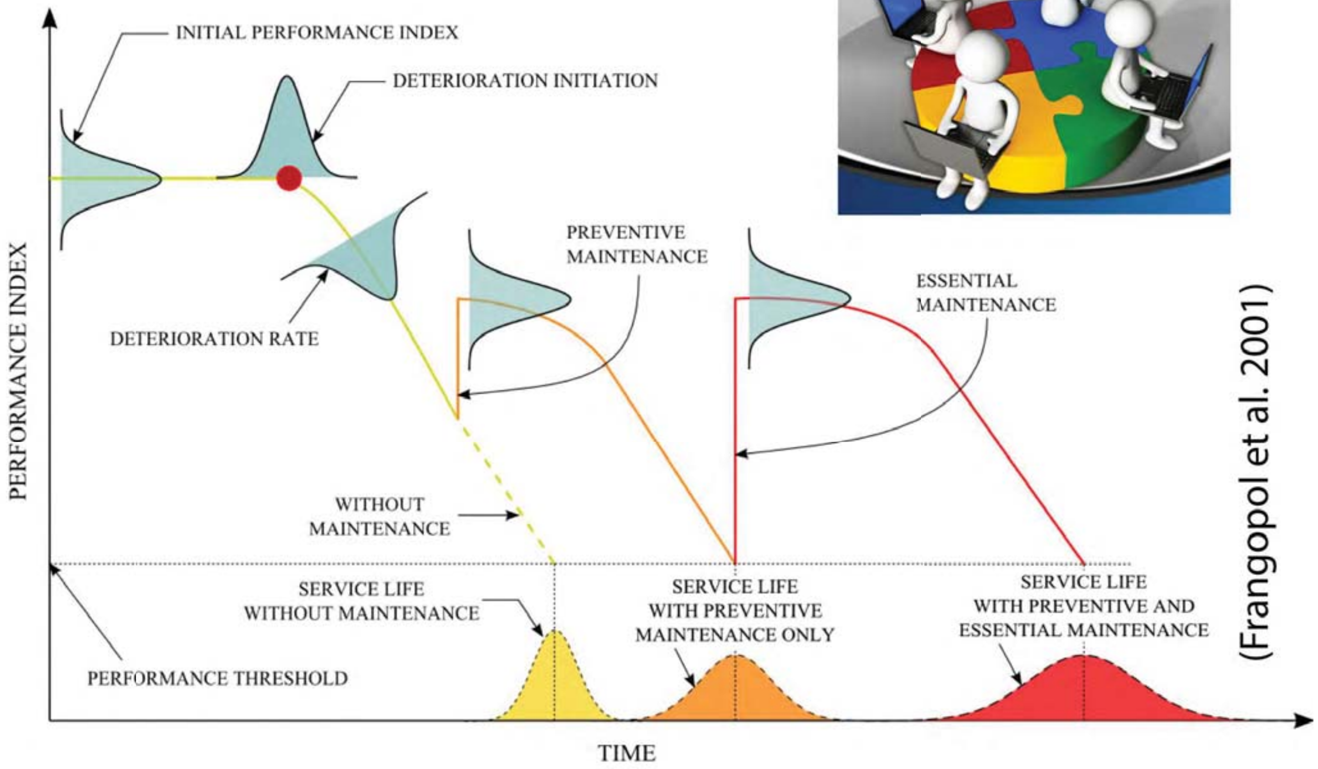


(source: xkcd blog)

Engineering Efficient Infrastructure

Repairing a Defective System





(Frangopol et al. 2001)

Exploiting Sensory feedback and appropriately Calibrated Computational models for Infrastructure Efficiency

The importance of Modeling

Main Challenges

- Incorporating Uncertainties
- Model Validation via Sensory Feedback
- Deliver accurate Reduced Order Models - Metamodels
- Increase Computational Efficiency at a lower Computational toll – the Multiscale formulation

Structural Health Monitoring

Sensory feedback via **SHM systems**.

Force

Displacement

Strain and tilt

Acceleration

Meteo

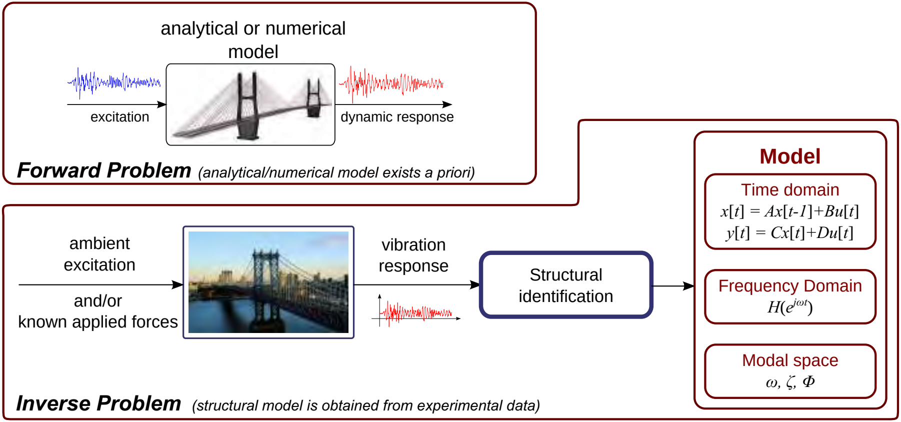
Data from GPS

System Identification in SHM

Link between acquired data & modeling of structural behavior?

System Identification

Developing or improving the mathematical representation of a physical system using experimental data.



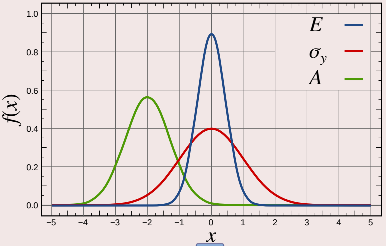
Reality Check

Challenge #1: Fusion of heterogeneous data, sensor noise

Reality Check

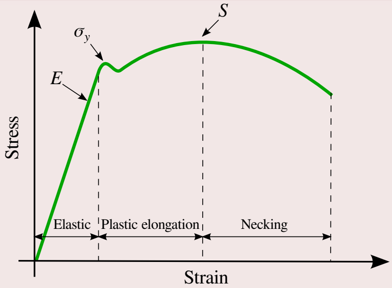
Challenge #2: Lack of a-priori knowledge of the system itself

Structural system is characterized by parameter uncertainty



The behavior of the modeled structure has to be examined for a range of structural characteristics.

Nonlinearities are taken into account



The impact of different types of excitation (of different magnitude and/or spectral content) should also be examined.

Optimal Bayesian Solution

The Task: How to estimate $\mathbf{x}$ given **partial, noisy observations** of the response $\mathbf{y}$?

Predict

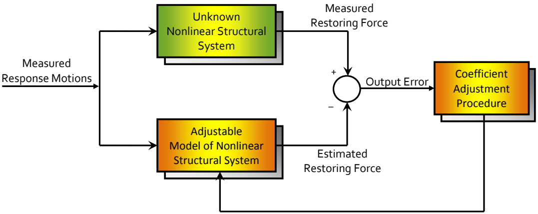
Assuming the prior $p(x_0)$ is known and that the required pdf $p(x_{k-1}|y_{1:k-1})$ at time $k-1$ is available (**Chapman-Kolmogorov equation**):

$$p(x_k|y_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|y_{1:k-1})dx_{k-1}$$

Update

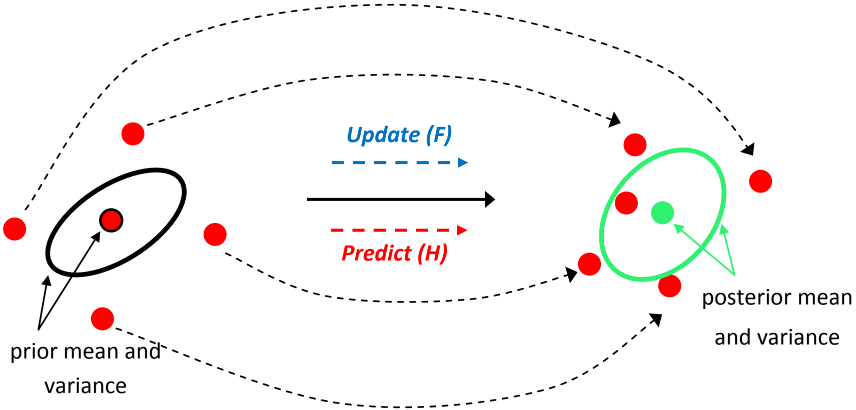
Consequently, the prior (or prediction) is updated using the measurement y_k at time k(**Bayes Theorem**):

$$p(x_k|y_{1:k}) = p(x_k|y_k, y_{1:k-1}) = \frac{p(y_k|x_k)p(x_k|y_{1:k-1})}{p(y_k|y_{1:k-1})}$$



Bayesian Approximation

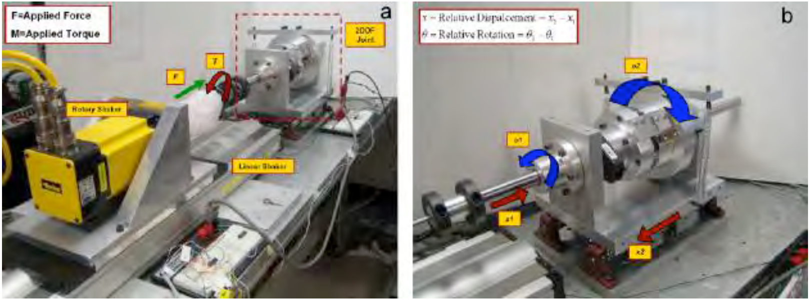
Particle-based Bayesain approaches - Working principle



Chatzi, E. N. and Smyth, A. W. (2012), "Particle filter scheme with mutation for the estimation of time-invariant parameters in structural health monitoring applications", Journal of Structural Control and Health Monitoring.

Application #1: Treating model Uncertainty in real-time

On Line Parametric Identification of a Non-Linear Hysteretic System with Model Uncertainty

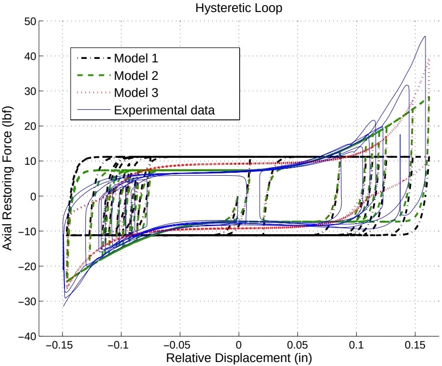


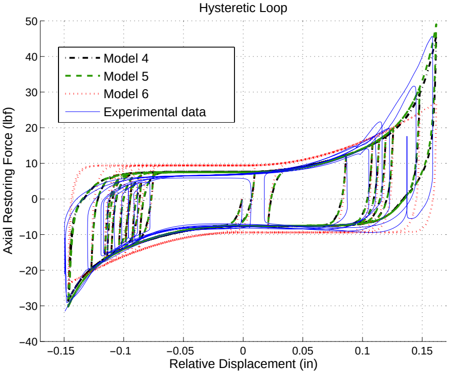
Experimental Setup at USC

E.N. Chatzi, A.W. Smyth and S.F. Masri, "Experimental application of on-line parametric identification for nonlinear hysteretic systems with model uncertainty", Journal of Structural Safety, Structural Safety, Vol. 32, No. 5. (24 September 2010), pp. 326-337.

Nonlinear Hysteretic Joint

Non typical hysteretic behavior



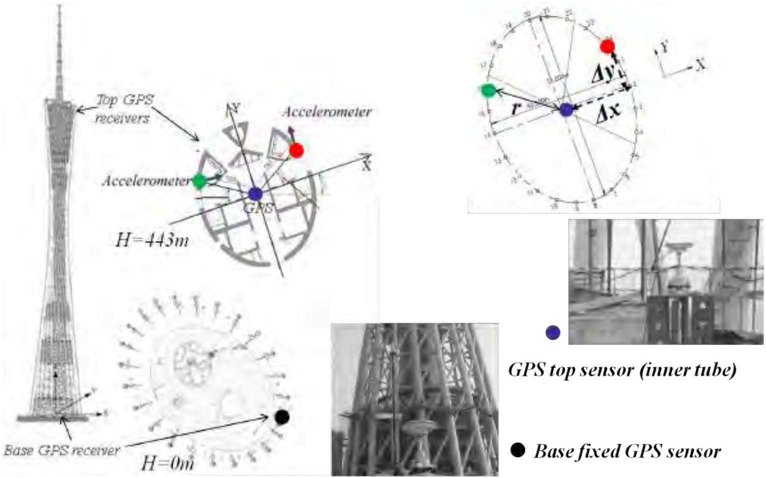


A physical model of the system is formulated **on-the-fly**:

$$\dot{z} = \mathscr{B}_1 \oplus \{c_1 \, |x|^{n_1} \, \dot{x}(\text{sgn}\,(x\dot{x})+1)/2 + c_2 \sinh\,(a_2 x \text{sgn}\,(\dot{x}))\,\dot{x}\}, \;\; l = 5$$

Application #2: Actual Large Scale Structure

Case Study - Tall Tower Structure



E. Chatzi, C. Fuggini, "Structural identification of a super-tall tower by GPS and accelerometer data fusion using a multi-rate Kalman filter", 3th International Symposium on Life-Cycle Civil Engineering (IALCCE 2012), October 3-6, 2012.

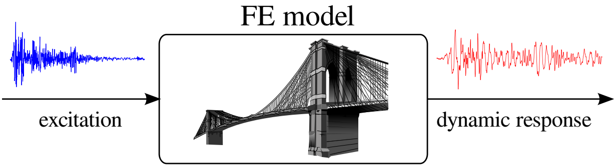
Quantifying Uncertainty

Challenge: How to Quantify Uncertainty through efficient computation?

Motivation

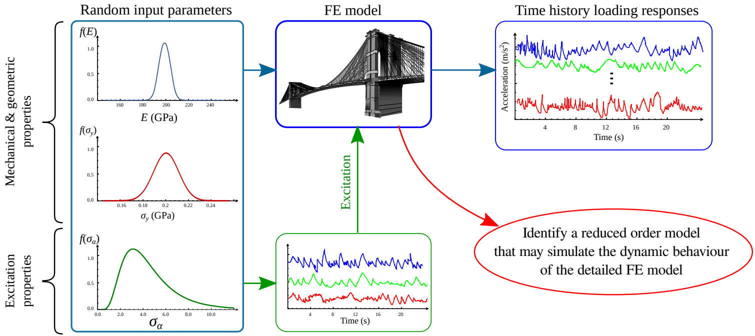
The simulation of dynamic response through FE models requires excessive computational resources particularly for complex, large structures. The problem is even more pronounced when:

- the structural system is characterized by parameter uncertainty
- detailed geometrical descriptions are adopted
- nonlinearities are taken into account



This is especially important for the case of inverse problem formulations or Reliability Analyses where a large number of forward runs is necessary.

One Step Further - The Metamodeling approach



Problem definition

Consider a structural system represented by a numerical model $\mathscr{M}$ characterized by uncertain input parameters $\xi = [\xi_1, \xi_2, \ldots, \xi_M]^{\mathsf{T}}$ with known joint pdf $f(\xi)$. The dynamic response of $\mathscr{M}$ to a given input excitation $x[t,\xi]$ will also be a random variable:

$$y[t,\xi] = \mathscr{M}(x[t,\xi],\xi), \quad t = 1,2,\ldots,T$$

A metamodel $\tilde{\mathscr{M}}$ must be able to predict and/or simulate the detailed numerical model results in a computationally inexpensive way and with sufficient accuracy.

PC-ARX Models

Polynomial Chaos AutoRegressive with eXogenous input (PC-ARX) models

$$\overbrace{y[t]+\sum_{i=1}^{n_a} a_i(\xi)\cdot y[t-i]}^{\text{AR part}} = \overbrace{\sum_{i=0}^{n_b} b_i(\xi)\cdot x[t-i]}^{\text{X part}} + e[t], \quad e[t]\sim \text{NID}(0,\sigma_e^2)$$

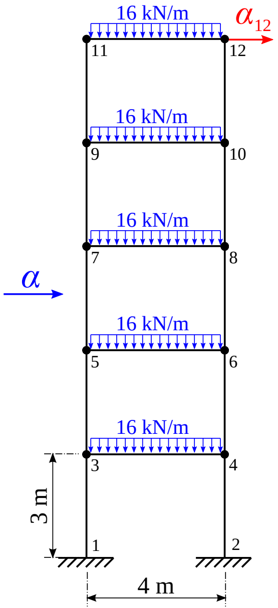
AR/X model parameters are modeled as random variables projected on a **polynomial chaos basis**, in order to enable uncertainty propagation.

$$a_i(\xi)=\sum_{j=1}^{p} a_{i,j}\cdot \phi_{d(j)}(\xi), \qquad b_i(\xi)=\sum_{j=1}^{p} b_{i,j}\cdot \phi_{d(j)}(\xi)$$

$a_{i,j}, b_{i,j}$: unknown deterministic coefficients of projection

$\phi_{d(j)}$: basis functions orthonormal w.r.t. the joint probability density function of ξ

Implementation on a 5 storey Nonlinear Frame



Simple Implementation Example

The described framework is implemented for the simulation of the response of a **five-storey shear frame**, subjected to a (known) dynamic input in the form of earthquake excitation.

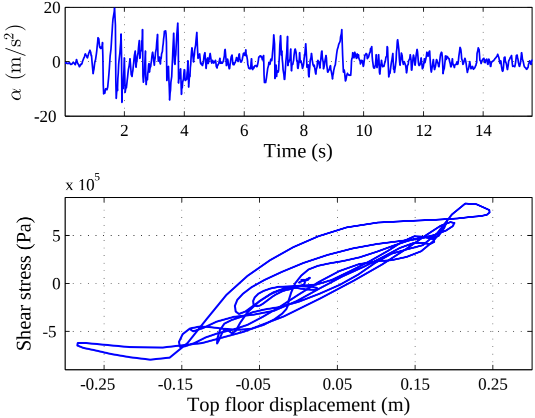
The frame is described by a nonlinear material law, allowing for the sections to move into the post-yield region which causes nonlinear behavior to occur.

We consider the following **input parameters**:

Input parameter	Vertical elements	Horizontal elements
Density (kg/m^3)	7850	7850
Poisson ratio	0.29	0.29
Young moduli (GPa)	$\mathscr{U}(190,210)$	$\mathscr{U}(190,210)$
Yield stress (MPa)	$\mathscr{U}(200,500)$	$\mathscr{U}(200,500)$
Cross section area (m^2)	$\mathscr{U}(0.04,0.09)$	0.0625

Implementation on a 5 storey Nonlinear Frame

One of the recorded acceleration instances for the El Centro earthquake* has been utilized as ground excitation:



causing the observed shear stress vs top floor displacement response.
The curve shown here corresponds to the first simulation experiment (with ξ_1) and $t = 1, 2, \ldots 250$.

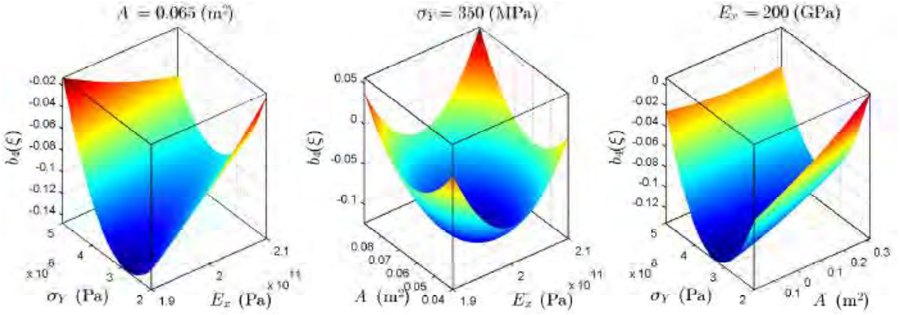
* downloadable at: http: //peer.berkeley.edu/peer_ground_motion_database

Implementation on a 5 storey Nonlinear Frame

20 simulations are conducted using a detailed structural model. The **ANSYS finite element software** has been used for the reference simulations.

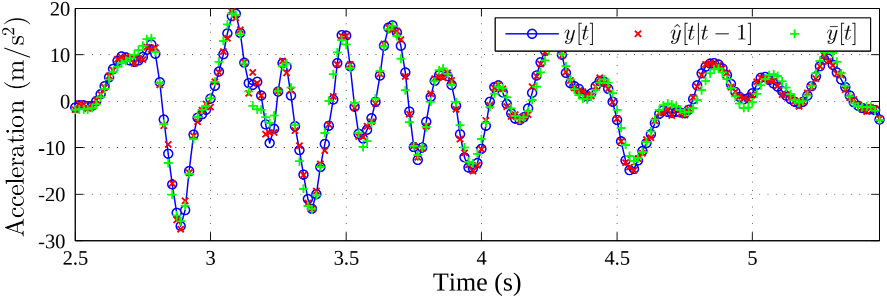
The derived functional representation

Polynomial expansion of $b_4(\xi)$ model parameter onto the input space



Implementation on a 5 storey Nonlinear Frame

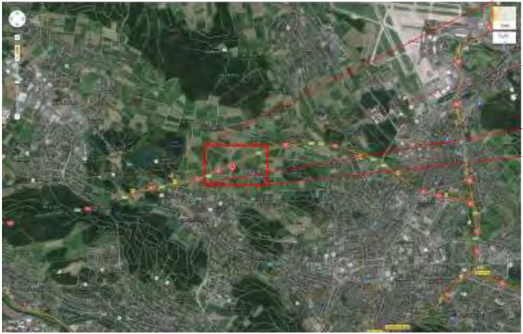
In order to **validate** the workings of the metamodeling framework the performance of the identified PC-ARX(10,10) metamodel is tested for the prediction and simulation of the dynamic response of the FE model subjected this time to the Pacoima Dam earthquake.



0.7836 % prediction error
3.7585 % simulation error
5000 times reduced simulation time

Running Field Project

Structural Identification for Condition Assessment of Swiss Bridges, Research Grant funded by the Federal Roads Office



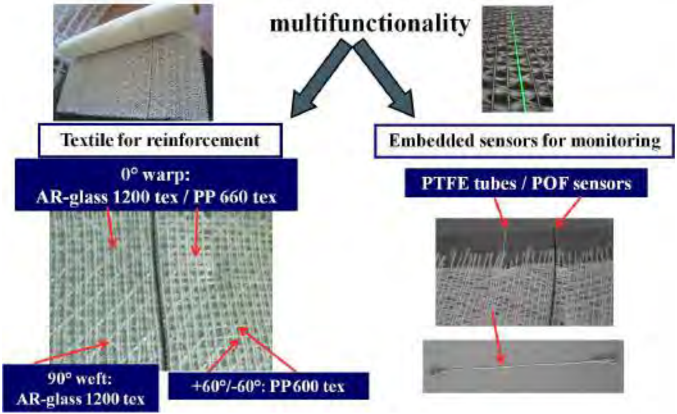
Structural Assessment of Complex Structures

Beyond Reduced Order Models it is often desirable to maintain model refinement at a reduced computational toll

Multi - Phase Structures

- Masonry structures constitute a large portion of the existing building stock
- Novel structures, largely based on composite & polymer materials are continuously emerging

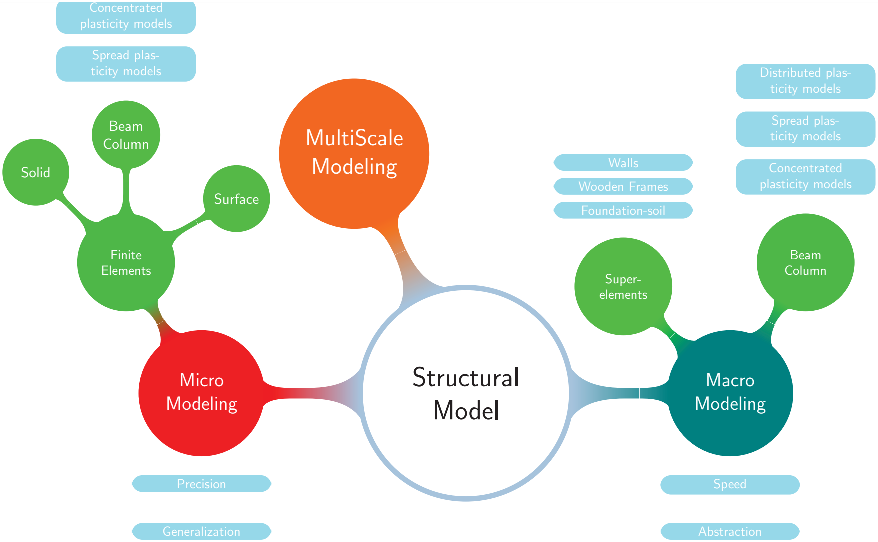
Implementing novel technologies in masonry retrofitting - SNSF project



Modeling Challenges

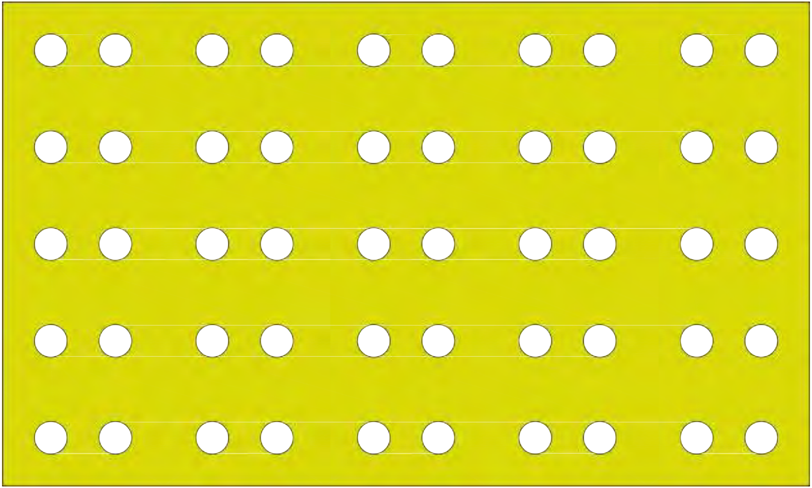
- Complex behaviour of the constituents - Crack propagation
- Material parameters hard to identify
- Standard FEM modeling procedures are too expensive

Structural Assessment of Complex Structures



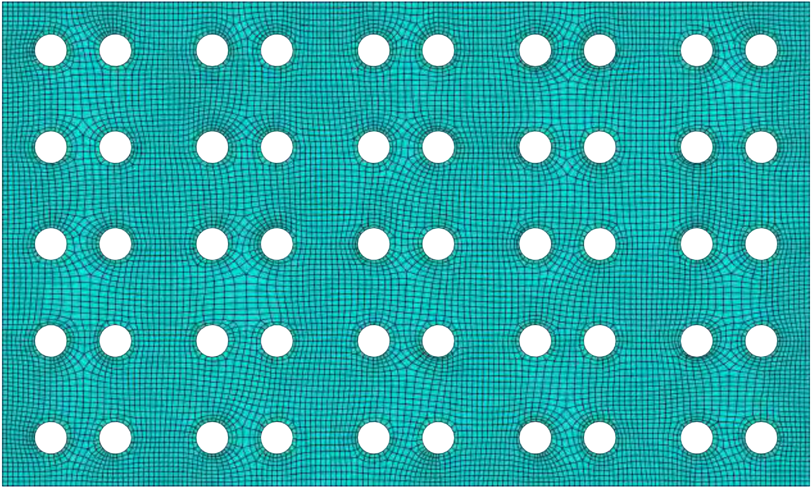
The Multiscale Finite Element scheme

Given a heterogeneous deformable body (flaws, inclusions, material layers)



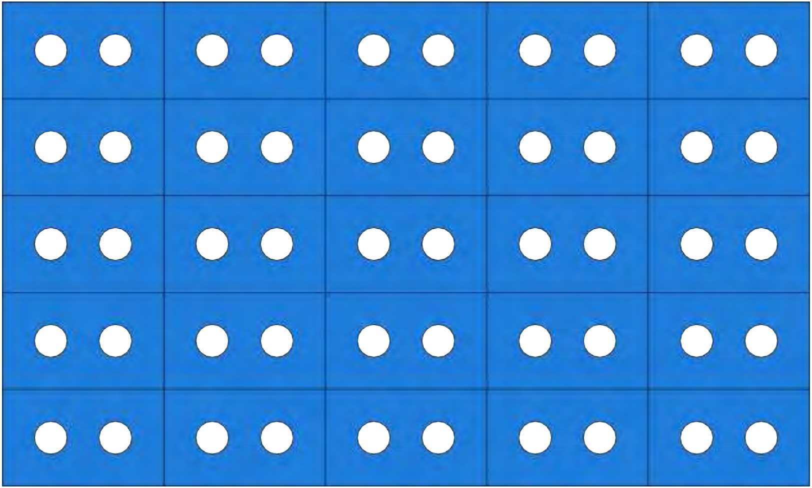
The Multiscale Finite Element scheme

And an accompanying fine mesh (standard FE approach)

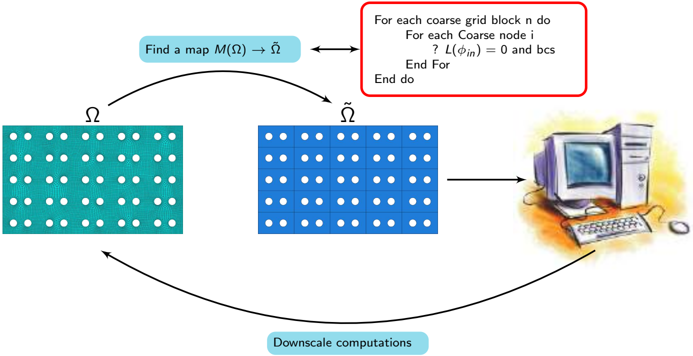


The Multiscale Finite Element scheme

Solve this mesh instead (multiscale approach)



MultiScale FEM



- Re-evaluation of the mapping is required in a nonlinear analysis
- Use the Hysteretic FE in the micro-scale

Proposed Approach:

Couple the Multiscale FE approach with a Hysteretic FE Formulation

The hysteretic formulation of Finite Elements

- Considering the additive decomposition of the strain vector

$$\{\dot{\varepsilon}\} = \{\dot{\varepsilon}^{e}\} + \left\{\dot{\varepsilon}^{pl}\right\}$$

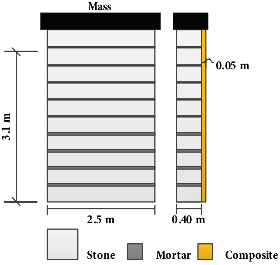
- An evolution equation for the plastic part of total strain is derived

$$\left\{\dot{\varepsilon}^{pl}\right\} = \left|\frac{\tilde{\Phi}}{\tilde{\Phi}_0}\right|^{N} \left(\beta + \gamma \mathrm{sgn}\left(\{\dot{\varepsilon}\}^{T}\{\sigma\}\right)\right)[R]\{\dot{\varepsilon}\}$$

Triantafyllou, S.P., Koumousis, V.K. (2012). "A Bouc-Wen Type Hysteretic Plane Stress Element", Journal of Engineering Mechanics, 138 (3), pp. 235-246

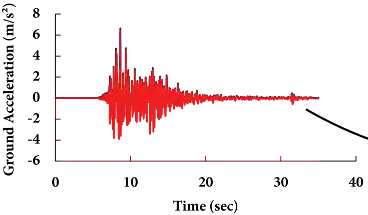
S. Triantafyllou and E. Chatzi (2013), "A novel Hysteretic Multiscale Finite Element Method for Nonlinear Dynamic Analysis of Heterogeneous Structures", 11th International Conference on Structural Safety & Reliability

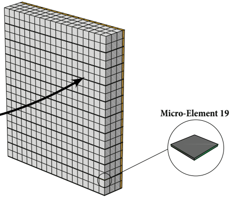
Textile Reinforced Masonry Wall



(a)

	Mortar	Stone
Young's modulus [MPa]	3494	20200
Poisson's ratio	0.11	0.2
Plasticity	Mohr-Coulomb	von-Mises
Friction angle [deg]	21.8	-
Cohesion [MPa]	0.1	-
Yield Stress [MPa]	-	69.2





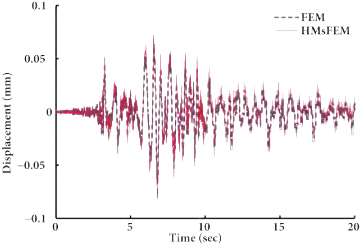
(b)

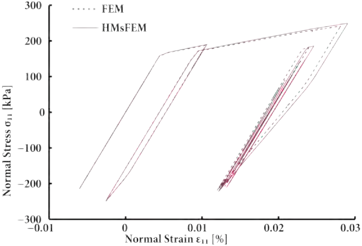
Composite Material)

Young's modulus [MPa]	$E_{11} = 54000$ $E_{12} = 53200$	$E_{22} = 53200$ $E_{23} = 54000$	$E_{33} = 53200$ $E_{23} = 54000$
Poisson's ratio	$\nu_{12} = 0.14$	$\nu_{23} = 0.2$	$\nu_{13} = 0.2$

Textile Reinforced Masonry Wall

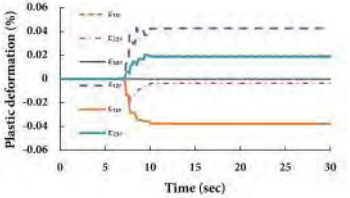
- Results derived from the HMsFEM formulation are compared to classical FEM

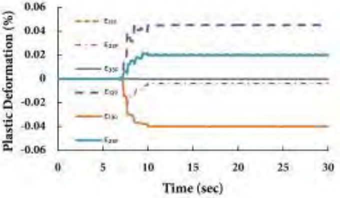




75% Reduction in Computational Time

- The plastic strain components are readily derived as part of the solution





Underlying System Model

- **Masonry** Material properties
- Constitutive Laws
- Structural Response under dynamic loading

Computational Tool

Multiscale Hys FEM

Applications

A computational analysis framework for dynamically evolving systems

Underlying System Model

- **Novel** Material properties
- Constitutive Laws
- Structural Response

Computational Tool

Multiscale Hys FEM

Ultra High Performance Fiber Reinforced Concrete

Applications

A **computational analysis** framework for **dynamically evolving** systems

Underlying System Model

- Bridge/Building Structural Model
- Material properties
- Loading Assumptions

Computational Tool

System ID Tools

Current Condition Assessment, Damage Detection & Quantification

Applications

Sensor Technology

A computational analysis framework for dynamically evolving systems

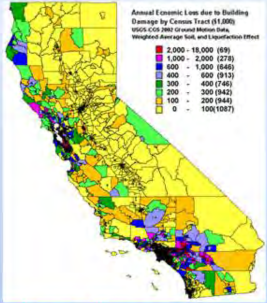
Underlying System Model

- Infrastructure Models
- Statistical Information
- Loading Assumptions

Computational Tool

UQ Tools

Sensory Information Grid



Quantification of damage on a «system» level

Applications

A computational analysis framework for dynamically evolving systems

Underlying System Model

- Traffic Models
- Statistical Information
- Loading Assumptions

Computational Tool

UQ Tools

Sensory Information Grid

Uncertainty Quantification on a Network level

Applications

A computational analysis framework for dynamically evolving systems

The Broad Picture

A gathering of Competences